A P Calculus Chapter 6 2002

We *love*

Applications

Pblm 35 pg 226 is longest pipe which can be taken around a corner. What does min for pblm 35 F prime test mean? etc.

Ch 6 Mech Exam

1 R/C pg 216 - 222 (skip example 5)
2 Pg 223 - 224 1 – 13
3 Pg 224 15, 17

11/22

Turn in Hw #1, 2

During Vacation:

4 Pg 225 19, 20
Write a very detailed argument explaining your reasoning for problems 19 and 20.

11/ 25

5 Pg 225 22, 27 - 31
Extend problem 22 with a Generalization by replacing the equation of the linewith: $y = mx + b$.

11/26

Thanksgiving Vacation

6 Pg 225 - 226
32, 33, 35
Use a calculator to solve the quadradic equation in problem 32

11/27

7 and #8
Read pgs 226 - 231
Then write a lecture covering that material.
#9
Pg 231 - 232 1 - 21, 23

11/28

10, 11
Rent, Borrow or whatever then **VIEW** the movie **The Legend of Bagger Vance**
Explain what happened to Jack Lemmon's character on the golf course in the very beginning of the movie and how it ties to the closing of movie.
One paragraph maximum

#12, 13, 14, 15, 16, "The Continuing Legend of Bagger Vance"

Have Hw 4, 5,and 6 put on boards and corrected.
Collect HW from vacation
Hand out
Bagger Vance

12 / 2

Put pblm 1-4 on boards

12 / 3

Put pblms
5 - 11 on boards

12 / 4

Put pblms
12 - 16 on boards

12 / 5

17
GQ C 6 pg 242 17, 18
GQ ch 1-6 Comp pg 243 10
GQ ch 1-6 Conc pg 244 7A
Collect Bagger Vance

12 / 6

18
Pg 245 - 246
27 - 29, 35ab, 40, 41, 43, 51

Review exercises

12 / 9

18 Pg 246 - 247
56, 57, 58, 60, 61, 64, 65
19
Pg 247 78 - 80

12/ 10

Put Last Year's Exam On The Chalkboard.

Come to class Prepared!

12/ 11

CALCULUS EXAM Chapter 6 "APPLICATIONS"

12/ 12

Grading Bagger Vance
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 16
17 18

Some Content from 183-191
Think current population

Given $f(t) = Ae^{kt}$ $f'(t) = kAe^{kt} = kf(t)$

$f'(t)$ is rate of growth; $kf(t)$ is proportional to Amt present

Question: what is e^k? (approximately)

* $e^k \approx 1+k$ for small k

if it's $e^k - 1 \approx k$

then $\frac{e^k-1}{k} \approx 1$

if * is true then

$\lim_{k\to 0} \frac{e^k-1}{k} = 1$

using l'Hopital's Rule

$\lim_{k\to 0} \frac{e^k}{1} = 1$ so

* is true for small k

"% increase" a definition

if 25 increased by 10% is

$25 + 25 \cdot 10\% =$

$25(1+10\%) = 27.5$

so A increased by k% becomes $A(1+k)$

Now $\frac{27.5}{25} = 1.1 = 1+10\%$

so $\frac{\text{New Amt}}{\text{Old Amt}} = 1 + \%$ growth

AT $t=0$ $f(0) = A$ $f(1) = Ae^k$

$\frac{f(1)}{f(0)} = \frac{Ae^k}{A} = e^k$ = growth per unit of time

But this ↑ yield $1+\%$ of growth so

$e^k = 1+\%$ of growth since * is true for small k then k is that % of growth

(1)

Conclusion: it is said that a "savings account grows at a rate of 8%". That is 8% is growth rate, under these conditions then we could say K is our growth Rate.

Summary:

Whether we call K the growth rate ( ) or not, our Book does call K "the growth rate (pg 183) so Beware.

Eg #1 pg 185

$t_0 \Rightarrow 1977 \Rightarrow 3.8$ Bill
growth rate 2%
find Population in 2000
$f(t) = Ae^{kt}$
$e^k = e^{.02} \approx 1 + .02 = 1.02$
$A = 3.8$
$t = 23$
$e^{kt} = (1.02)^{23}$

$f(23) = 3.8(1.02)^{23}$

≈ 6 Billion

(using calculator

Pg 208-209

① $f(x) = x^3 - 2x^2 + 5x - 1$

a) $f'(x) = 3x^2 - 4x + 5$

$f''(x) = 6x - 4$

$f'''(x) = 6$

$f^{iv}(x) = 0$

b) $f^{iv}(x) = 0 \quad \forall x \geq 4$

② $y = \sin 3x$

a) $y' = 3\cos 3x$

$y'' = -9\sin 3x$

$y''' = -27\cos 3x$

$y^{iv} = 81\sin 3x$

$y^{v} = 243\cos 3x$

③ $y = \tan^{-1}(2x)$

$y' = \frac{2}{1+4x^2} \qquad y'' = \frac{-2(8x)}{(1+4x^2)^2} = \frac{-16x}{(1+4x^2)^2}$

$y''' = \frac{-16(1+4x^2)^2 + 16x \cdot 2(1+4x)(8x)}{(1+4x^2)^4}$

$y''' = \frac{-16(1+4x^2) + 256x^2}{(1+4x^2)^3} = \frac{192x^2 - 16}{(1+4x^2)^3}$

④ $y = x\ln x - x$

$y' = \ln x + 1 - 1 = \ln x$

$y'' = \frac{1}{x}$

$y''' = \frac{-1}{x^2}$

$y^{iv} = \frac{+2}{x^3}$

(10) Let $\theta = f(t)$ is the θ in terms of time. Since the rate of change of the θ is changing more rapidly than

$f''(t) > 0$. If θ is decreasing $f'(t) < 0$

the "more rapidly" statement is the key.

(11) $y = ae^x + be^{-x}$

$y' = ae^x - be^{-x}$

$y'' = ae^x + be^{-x} = y$

(12) $y = a\sin kt + b\cos kt$

$y' = ak\cos kt - bk\sin kt$

$y'' = -ak^2\sin kt + bk^2\cos kt$

$= -k^2(a\sin kt + b\cos kt)$

$= -k^2 y$

(13) $y = (t-1)^{2/3}$

$y' = \frac{2}{3}(t-1)^{-1/3}$

$y'' = \frac{-2}{9}(t-1)^{-4/3} = -\frac{2}{9}\frac{1}{((t-1)^{2/3})^2} = -\frac{2}{9}\frac{1}{y^2}$

(14) obvious $v|_{t=0} = v_0$

(15) Since $v = -32t + C_1$ and at $t=0$ $64 = -32(0) = C'$
it C' still is 64
however
$y = -16t^2 + 64t + K$ changes C

$y = -16t^2 + 64t + 0$ is $-16t^2 + 64t$
since K is initial position

(16) it when $y = 96$ $y = -16t^2 + 64t + 96$

$-16t^2 + 64t + 96 = 96$
$t^2 - 4t = 0$
$t(t-4) = 0$
$t = 0$ $t = 4$
↑ INITIAL condition ↑ this is when it passes the edge of the cliff after being thrown
$v|_{t=4} = -32t + 64 = -128 + 64 = -64$ ft/sec
is falling 64 ft/sec.
Speed is 64 ft/sec. (Absol. value of velocity)

(17) $y = -16t^2 + 96$ is initial velocity will be zero
$0 \overset{\text{set}}{=} -16t^2 + 96$
$t^2 = 6$
$t = \sqrt{6}$

(18) $y = -16t^2 + 64t + 96$ ← INITIAL height.
↑ caused by gravity ↑ initial velocity & direction

(bottom pg 205)

(19) $2-\sqrt{10} < 0$ ∴ Before $t=0$. This is time the ball was thrown where height of ball is zero. when ball gets to height of cliff time will be zero.

(20) Let a = constant acceleration and $f(\tau)$ be position at time τ. $f(0)=0$

$f''(\tau) = a \Rightarrow f'(\tau) = a\tau + C$

but $f'(0)=0$ (its starts stationary)

So $C=0$ So $f'(\tau) = a\tau$

$f(t) = \frac{a\tau^2}{2} + C$ but again $C=0$ So

$f(\tau) = \frac{a\tau^2}{2}$

Now we are given

$$f'(15) = 15a = 88 \quad \therefore a = \frac{88}{15}$$

$$f(15) = \frac{1.88}{2.15}(15)^2 = 44.15 \qquad \begin{array}{r} 60 \\ 600 \end{array}$$

660 ft. traveled.

(21) $f''(t) = a \overset{\text{Given}}{=} -16$

$f'(t) = -16t + K \qquad f'(0) = 88 \therefore K = 88$

$f'(t) = -16t + 88$

$f(t) = -8t^2 + 88t + K \qquad f(0) = 0 \therefore K = 0$

$f(t) = -8t^2 + 88t$

$= -8t^2 + 88t$

at 60 mph (88 FT/Sec) we traveled $88 \cdot \frac{6}{10}$ feet $\approx$ 52.8' before we hit brake.

$f'(t) \overset{\text{set}}{=} 0 = -16t + 88$

$t \approx 5.5$ when velocity is zero we have stopped.

$f(5.5) \approx 242$ dist after begin braking

$\therefore$ total dist $242 + 52.8 = 294.8$ ft.

(Actual dist $\quad r$ is reaction time (in seconds)

$d(t) = -16t^2 + (r + t)(\text{speed ft/sec}))$

? (21) $A(t) = -16t^2$

dist
speed
acceleration

Redone Previous Page.

$f''(t) = -16$

$f'(t) = -16t + V_0 \overset{\text{Set}}{=} 0$ $\quad t = \frac{60}{16}$ $\quad \frac{15}{4}$ $\quad \frac{15}{4} = 3.75 = 6$

$f(t) = -8t^2 + V_0 t$ $\quad -8(3.15)^2 + 60(3.15) =$

$-8t^2 + 60t \overset{\text{Set}}{=} 0$ $\quad 79.38 \quad 189$

$-4(2t^2 + 15t) = 0$

$t(2t + 15) = 0$

$t = \frac{-15}{2}$

(22) Let $f(t)$ = height of ball at time t.

Let $f(0) = 0$ (initial height is zero)

Let $f'(0) = V_0$ initial Velocity of Throw

Let $f''(0) = a_0$ initial acceleration (or deceleration)

$f''(t) = -32$ So $f'(t) = -32t + V_0$

$f(t) = -16t^2 + V_0 t + C$ but $f(0) = 0$ so

$f(t) = -16t^2 + V_0 t$ $\quad f'(t) = -32t + V_0 \overset{\text{Set}}{=} 0$ Time t max height

$t = \frac{V_0}{32}$

$f(t) \overset{\text{Set}}{=} 0 = -16t^2 + V_0 t \overset{\text{Set}}{=} 0$

$t(-16t + V_0) = 0$

$t = \frac{V_0}{16}$ Time from throw to Even with throw. exactly $\frac{1}{2}$ of time to max height.

Velocity when it hits $f'\left(\frac{V_0}{16}\right) = -32 \cdot \frac{V_0}{16} + V_0 = -V_0$

is the speed is the same as the initial Velocity.

eg want 100 items a yr, will order 5 times
∴ 20 items per order

P is a constant charge per item ∴
Px = cost relating to size of order

"minimize costs as a fctn of the size of the order"

∴ let x = # of units in each order
and COA is item to be minimized

Applied max min pblms. 6.8
Pg 216–226
Lecture

eg 1, 2, 3 Routine.

eg 4. Notice how they ck for global min.
(bottom of Pg 224)

eg 5. Not Routine

Let x = # of units in EACH order.

C(x) = Total annual carrying & reorder costs when firm orders x units in each lot.

A = total # of units bought for year.

∴ firm orders $\frac{A}{x}$ times a year "frequency"

Let f = fixed cost for placing 1 order. (phone call) (fixed amt)

P·x = Costs relating to size of order (shipping)

f + P·x = Total cost per order. So

$(F + P \cdot x)\frac{A}{x}$ = Total expense for ordering throughout the year.

(F + P·x) = cost per order; $\frac{A}{x}$ = # of orders

reorder costs

we want to minimize this

Notice: when x → 0 cost go up. if the cost to order smaller lots is higher.

"carrying inventory"

Let I = Carrying costs such as storage & inventory tax per yr. per unit.

CARRYING COSTS

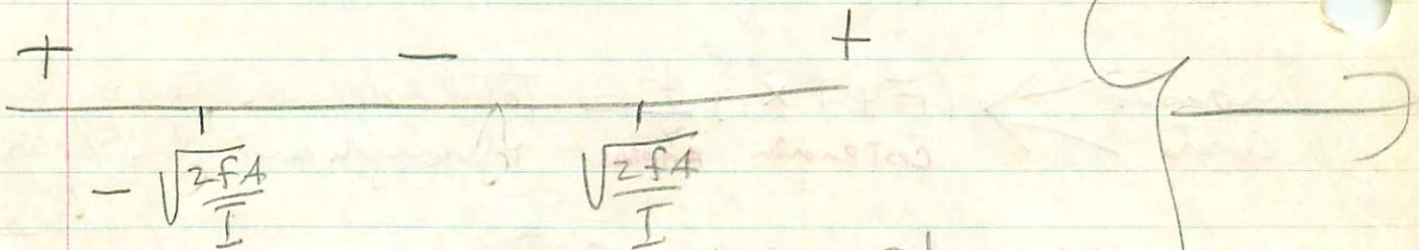
when the firm orders x units at a time its average inventory is $\frac{x}{2}$ so the total carrying costs per yr are $I \cdot \frac{x}{2}$

Notice when x is small so is carrying costs.

cont'd ↓

$$x^2 - \frac{2fA}{I} = 0$$

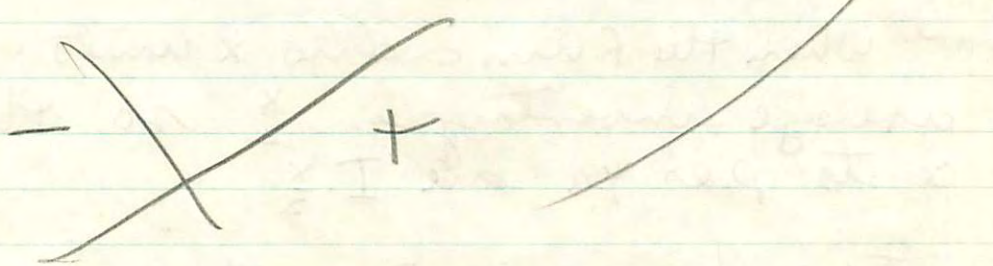
$$\left(x - \sqrt{\frac{2fA}{I}}\right)\left(x + \sqrt{\frac{2fA}{I}}\right) = 0$$



if if $|x| < \sqrt{\frac{2fA}{I}}$ then $f' < 0$

else

$f' > 0$



Now $C(x)$ is the fctn describing the total reorder & carrying costs so

$$C(x) = \left(f + Px\right)\left(\frac{A}{x}\right) + I \cdot \frac{x}{2}$$

$$C(x) = \frac{Ix}{2} + \frac{fA}{x} + PA$$

$$C'(x) = \frac{I}{2} - \frac{fA}{x^2}$$

$$= \frac{Ix^2 - 2fA}{2x^2} \overset{\text{set}}{=} 0$$

$$x^2 = \frac{2fA}{I} \Rightarrow x = \sqrt{\frac{2fA}{I}}$$

using f'tst $\quad x < \sqrt{\frac{2fA}{I}} \quad f' < 0$

$\quad x > \sqrt{\frac{2fA}{I}} \quad f' > 0$

$\therefore$ min at $x = \sqrt{\frac{2fA}{I}}$

To minimize costs the firm should order $x = \sqrt{\frac{2fA}{I}}$ units at a time.

Now the math job is complete. The company may or may not order $\sqrt{\frac{2fA}{I}}$ units at a time depending on:

if $A < \sqrt{\frac{2fA}{I}}$ (very small) company would simply order 1 (or 15 supply)

Note - in $\sqrt{\frac{2fA}{I}}$ A is the only variable... any change in A would be reflected in a proportional change in $\sqrt{A}$ $\left(\sqrt{\frac{2F}{I}}\right)$.

Lecture
P. 223

order size

$x = \sqrt{\frac{2fA}{I}} \Rightarrow$

frequency is # of orders per year would be $\frac{A}{x}$ so $\frac{A}{\frac{\sqrt{2fA}}{\sqrt{I}}} =$

$\frac{A}{x} = \frac{A\sqrt{I}}{\sqrt{2fA}} = \sqrt{\frac{A^2 I}{2fA}} = \sqrt{\frac{AI}{2f}} =$ frequency.

what if A increased 4 times? 3 choices.

① order 4 times as often the same quantity

② increase lot size 4 times & keep same order period

③ strike a balance between ① & ②.

frequency is $\sqrt{\frac{I}{2f}}\sqrt{A}$. if A quadruples

then $\sqrt{\frac{I}{2f}}\sqrt{4A} = 2\sqrt{\frac{I}{2f}}\sqrt{A} \Rightarrow$ double frequency.

because the order size is proportional to $\sqrt{A}$ we would double the frequency of orders. & those orders would have lot size !

$x = \sqrt{\frac{2f(4A)}{I}} \Rightarrow 2\sqrt{\frac{2fA}{I}} =$ double in size

Pg 223-224

1-6 Routine

(7) $f(x) = 18x^2 - x^3$ this describes mass

a) $f'(x) = 36x - 3x^2$ this is density.

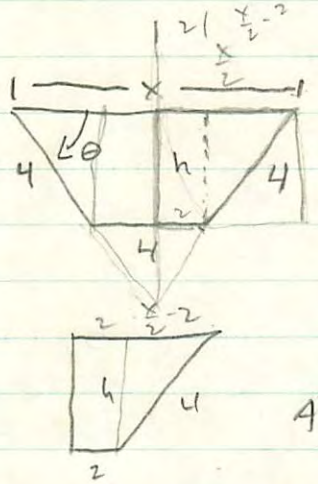
b) to locate max density we must find max of $f'(x)$.

$f''(x) = 36 - 6x \overset{\text{Set}}{=} 0$

$x = 6$ if $x < 6$ $f'' > 0$ if $x > 6$ $f'' < 0$

$\therefore$ f' test (as applied) shows max at $x = 6$.

(8)



A) Area of isoceles TRAPezoid. $\frac{1}{2}(a+b)h$

$A(x) = \frac{1}{2}(x+4)(h)$ $\qquad h^2 = 16 - \left(\frac{x}{2} - 2\right)^2$ $\qquad \frac{x-4}{2}$

$h = \frac{\sqrt{64 - (x^2 - 8x + 16)}}{2}$

$A(x) = \frac{1}{4}(x+4)\sqrt{48 + 8x - x^2}$

$\left(\frac{x-4}{2}\right)^2 = \frac{1}{2}\sqrt{48 + 8x - x^2}$

$A'(x) = \frac{1}{4}\sqrt{48+8x-x^2} + \frac{1}{4}(x+4)\frac{1 \cdot (8-2x)}{2\sqrt{48+8x-x^2}}$

$\frac{x^2 - 8x + 16}{4}$

$A'(x) = \frac{48 + 8x - x^2 + (4+x)(4-x)}{4\sqrt{8x - x^2}}$

$\sqrt{8x - x^2}$

$A'(x) = 48 + 8x - x^2 + 16 - x^2$

$\frac{4-x}{\frac{1}{2}(8-2x)}$

$-2x^2 + 8x + 64$

$\frac{1}{2}\sqrt{8x-x^2}$

$\frac{-2(x^2 - 4x - 32)}{4\sqrt{8x - x^2}} = \frac{(-)(x+4)(x-8)}{2\sqrt{8x - x^2}} \overset{\text{Set}}{=} 0$

Notice neg. don't eliminate it!

$x = 8,$

- 0 + 0 −

-4 0 8

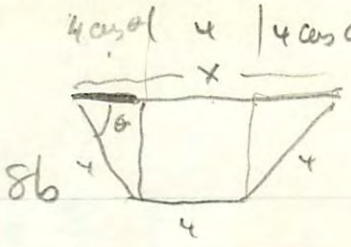
& B Next Pg!

~~Also~~

~~Trapezoid~~

~~Since this is a trapezoid~~

max f' ~~test~~.



8b

$A = \frac{1}{2}(x+4)h$ $\quad h = 4\sin\theta$

~~$A = \frac{1}{2}(x+4)\sin\theta$~~ $\quad x = 8\cos\theta + 4$

$A = \frac{1}{2}(8\cos\theta + 8)4\sin\theta$

$A = 16(\cos\theta + 1)\sin\theta$

$A' = -16\sin^2\theta + 16\cos\theta(\cos\theta + 1)$

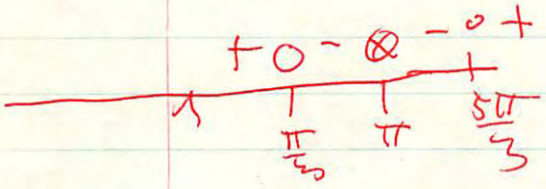
$= -16(1-\cos^2\theta) + 16\cos^2\theta + 16\cos\theta$

$-16 + 16\cos^2\theta + 16\cos^2\theta + 16\cos\theta$

$= 32\cos^2\theta + 16\cos\theta - 16$

$= 16(2\cos^2\theta + \cos\theta - 1)$

$16(2\cos\theta - 1)(\cos\theta + 1)$



← non participating pt.

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

interval $0 \le \theta \le \pi$

8c) Area found in 8a $A(8) = \frac{1}{4}(8+4)\sqrt{48+64-64}$

$3\sqrt{48} = 12\sqrt{3}$

Area found in 8b $A(\frac{\pi}{3}) = 16(\frac{3}{2})\frac{\sqrt{3}}{2} = 12\sqrt{3}$

both areas are =.

there can only be one max area. No matter how it is measured.

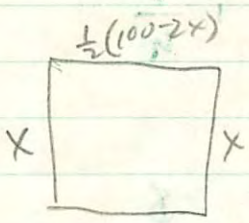
(9) $h(t) = -16t^2 + 40t$

a) $h'(t) = -32t + 40 \overset{\text{set}}{=} 0$

$t = \frac{40}{32} = \frac{5}{4}$ 1 1/4 sec.

b) $h\left(\frac{5}{4}\right) = -16 \cdot \frac{25}{16} + 40 \cdot \frac{5}{4} = 25.$

(10)



$A(x) = x(50-x) = 50x - x^2$

$A'(x) = 50 - 2x \overset{\text{set}}{=} 0$

$x = 25$; it Square.

(11) $R(p) = p(9-p^2) = 9p - p^3$ (Unit price · Volume)

$R'(p) = 9 - 3p^2 \overset{\text{set}}{=} 0$

a) \$1,73 (ea) $p = \sqrt{3}$ opt price for max Revenue.

$R(\sqrt{3}) = 9\sqrt{3} - 3\sqrt{3} = \$6\sqrt{3}$ thousands $\approx$ \$10,39 thousand

\$10,392.30

$\frac{\sqrt{11} \pm 1}{2}$

$x = \frac{-1 \pm \sqrt{11}}{2}$

b) Let 1st price $= x$ — price $9 - x^2$

2nd price $= x+1$ — price $9 - (x+1)^2$

$R(x) = x(9-x^2) + (x+1)(9-(x+1)^2)$

$= 9x - x^3 + (x+1)(8 - 2x - x^2)$

$= 9x - x^3 + 8 + 6x - 3x^2 - x^3$

$8 - 2x - x^2$

$x + 1$

$8 - 2x - x^2$

$8x - 2x^2 - x^3$

$R(x) = -2x^3 + 3x^2 + 15x + 8$

$R'(x) = -6x^2 - 6x + 15$

$-3(2x^2 + 3x - 5) \overset{\text{set}}{=} 0$

$\frac{-2 \pm \sqrt{4+40}}{4}$

$\frac{-2 \pm 2\sqrt{11}}{4}$

$\frac{-1 \pm \sqrt{11}}{2}$ reject $\frac{-1-\sqrt{11}}{2}$

$x = \frac{-1 + \sqrt{11}}{2} \Rightarrow \$1.16,$

at this price = \$1 more

\$2.16

→2(1,561) =

→3(1,3456)

+17.4

25.4

18.24 thousand

\$18,241.41

$$\frac{1}{100}\left(5+\frac{s}{10}\right)\cdot 600 \;+\; \frac{600}{s}\cdot\frac{18}{s}$$

$$\frac{\$}{\text{cents}}\cdot\frac{\text{cents}}{\text{mile}}\cdot\text{miles} \qquad \frac{\text{miles}}{\frac{\text{miles}}{\text{hr}}}\cdot\frac{\$}{\text{hr}}$$

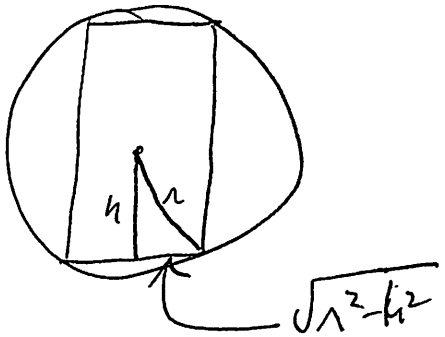
\$ for Truck

$$\frac{\text{hr}}{1}\cdot\frac{\$}{\text{hr}}$$

\$ driver

$C(s) =$

Stiffness $= (h^3 w) K$



$$f(h) = (2h)^3 2\sqrt{r^2-h^2}$$

$$16h^3\sqrt{r^2-h^2}$$

$$f'(h) = \sqrt{r^2-h^2}\cdot 48h^2 + \frac{16h^3 \cdot -2h}{2\sqrt{r^2-h^2}} \overset{SET}{=} 0$$

$$\frac{48h^2(r^2-h^2) - 16h^4}{\sqrt{r^2-h^2}}$$

$$\frac{48h^2r^2 - 48h^4 - 16h^4}{\sqrt{\quad}} \overset{SET}{=} 0$$

$$48h^2r^2 - 64h^4$$

$$16h^2(3r^2 - 4h^2) = 0$$

$$4h^2 = 3r^2$$

$$h^2 = \frac{3}{4}r^2$$

$$h = \frac{r\sqrt{3}}{2}$$

$$\sqrt{r^2 - \frac{3}{4}r^2} = \frac{r}{2}$$

so height is $2 \cdot \frac{r\sqrt{3}}{2} = r\sqrt{3}$

and width is $2 \cdot \frac{r}{2} = r$

$$\begin{array}{ccc} + & 0 & - \\ \hline & \uparrow & \\ 0 & & \frac{r\sqrt{3}}{2} \end{array}$$

600 miles / s mi/hr.

(12) cents. Cost/mile $\left(5 + \frac{s}{10}\right) \cdot 60$

a) You would want s to be smaller ie lower cost/mile

b) You want s large ie fewer hrs.

c) $ / Mile hrs $/hr.

Cost $\frac{1}{100} 600\left(\frac{50+s}{10}\right) + \frac{600}{s} \cdot \frac{18}{5}$

$= \frac{6}{10}(50+s) + \frac{2160}{s}$ s miles/hr.

$C(s) = 30 + \frac{3}{5}s + \frac{2160}{s}$

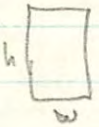
$= \frac{+3}{5} - \frac{2160}{s^2}$

$\frac{3s^2}{5} = 2160$

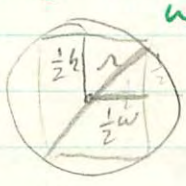
$s^2 = 3600$

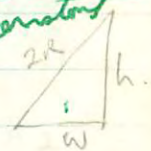
$s = 60$ OPT speed is 60 mph.

(13)



Stiff $= (h^3 w)k$ ← is proportional to ie diff wood has diff constant





$S = h^3\sqrt{4R^2 - h^2}$

$S' = 3h^2\sqrt{4R^2-h^2} - \frac{2h^4}{2\sqrt{4R^2-h^2}}$

r is a constant

$= \frac{3h^2(4R^2-h^2) - h^4}{\sqrt{4R^2-h^2}}$

do problem in h since it is cubed & the binomial would be too difficult if cubed.

$= \frac{12h^2R^2 - 3h^4 - h^4}{\sqrt{\ }} = \frac{12h^2R^2 - 4h^4}{\sqrt{\ }}$

$= \frac{4h^2(3R^2 - h^2)}{\sqrt{\ }} \overset{\text{set}}{=} 0$ $h^2 = 3R^2$

$h = R\sqrt{3}$

$w = \sqrt{4R^2 - 3R^2} = R$

Dimensions of Beam $R \times \sqrt{3}R$.

(14) Press cost $\left(\frac{200,000}{1,200}\right)2$ using 1 Plate.

Let N = # of Plates.

$$C(N) = \left(\frac{1000}{3}\right)\frac{1}{N} + \frac{55}{100}N$$

$$= \frac{1000}{3N} + \frac{11N}{20}$$

$$C' = -\frac{1000}{3N^2} + \frac{11}{20} \overset{\text{set}}{=} 0$$

$$20000 = 33N^2$$

$N = 24.6$ ∴ use 25 plates.

(15)

$$H(P) = -P\ln P - [(1-P)\ln(1-P)]$$

$$H'(P) = -\ln P - 1 + [-\ln(1-P) - 1]$$

$$-\ln P - 1 + \ln(1-P) + 1$$

$$\overset{\text{set}}{=} \ln(1-P) - \ln P \Rightarrow \frac{1}{1-P} - \frac{1}{P} = \frac{-1}{P(1-P)}$$

$$0 \overset{\text{set}}{=} \ln\left(\frac{1-P}{P}\right)$$

$$1 = \frac{1-P}{P} \qquad H'' = \frac{P}{1-P}\left(\frac{-P-(1-P)}{P^2}\right)$$

$$P = 1-P \qquad H'' = \frac{-1}{P(1-P)}$$

$$2P = 1$$

$$P = \frac{1}{2} \qquad H''\left(\frac{1}{2}\right) < 0 \quad \therefore \text{ max } f'' \text{ test.}$$

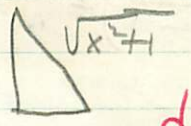
Point out the Necessity of Unifying using the proper test and

require "meaningful, complete sentences"

(19) W to show

/ is cheaper than L

if $s < 3/4$ question, is



direct cost down & across cost

is $\left(\sqrt{x^2+1}\right)(5280)(\$5) < 5280(\$3 \cdot x + \$5 \cdot 1)$

$5\sqrt{x^2+1} < 3x+5$

Define $f(x) = 5\sqrt{x^2+1} - 3x - 5$ if $f > 0$ $\forall x$ $(0, \frac{3}{4})$ then direct is cheapest.

$f(0) = 0$ $\quad f'(x) = \frac{5 \cdot 2x}{2\sqrt{x^2+1}} - 3$

$\frac{5x - 3\sqrt{x^2+1}}{\sqrt{x^2+1}}$

$\frac{9}{16}+1$

$3\frac{5}{4}$

$\frac{15}{4}$

we need to show $f'(x) > 0$ $\forall x$ in the interval

$f'(0) = 2$ $\quad f'(\frac{3}{4}) = \frac{5 - 3\sqrt{\frac{9}{16}+1}}{\sqrt{\frac{9}{16}+1}} = \frac{5-3\sqrt{\frac{25}{16}}}{\frac{5}{4}} =$

(since f' stays >0 we can say f is an increasing fct N.) NO. $= \frac{5 - \frac{15}{4}}{\frac{5}{4}} = \frac{\frac{5}{4}}{\frac{5}{4}} = 1$

That is, for this interval f increases from $f(0) = 0$ to f is >0

$\frac{5x - 3\sqrt{x^2+1}}{\sqrt{x^2+1}} \overset{\text{set}}{=} 0$ $\quad 25x^2 = 9(x^2+1)$

$16x^2 = 9$

$5x = 3\sqrt{x^2+1}$ $\quad x = \frac{3}{4}$ $\quad (4x-3)(4x+3)$

$\frac{25}{4} = x^2+1$

$x =$

f' + 0 − 0 +

$-\frac{3}{4}$ $\quad 0$ $\quad \frac{3}{4}$

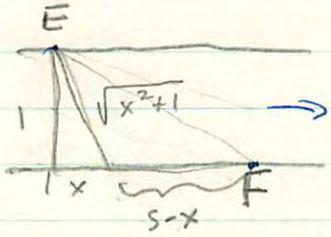
in interval $(0, \frac{3}{4})$ $f' < 0$ $\therefore$ f is decreasing — fm zero

(17) $y = \frac{cs}{4} + \frac{R}{vs}$ $\quad y'' = \frac{2R}{vs^3} > 0$

$y' = \frac{c}{4} - \frac{R}{vs^2} \overset{\text{set}}{=} 0$ $\quad \therefore$ min at Any critical value is zero of y'.

$\frac{R}{vs^2} = \frac{c}{4} \Rightarrow \frac{R}{vs} = \frac{cs}{4}$

(19)



land $3/foot $5/water.

$0 \le x \le s$

a) all water if s very small if s very large straight across river then along land. You could reflect as in geometry.

b) $C(x) = (5280)5\sqrt{x^2+1} + 3(5280)(s-x)$

$C'(x) = \frac{(5280)5(x)}{x\sqrt{x^2+1}} - 3(5280)$

$= \frac{(5280)5x - 3(5280)\sqrt{x^2+1}}{\sqrt{x^2+1}} \overset{\text{set}}{=} 0$

$5x - 3\sqrt{x^2+1} = 0$

$25x^2 = 9x^2 + 9$

$16x^2 = 9$

$x = \frac{3}{4}$

$C(\frac{1}{4}) = 5280 \cdot 5 \cdot \frac{\sqrt{17}}{4} + 3(5280)(\frac{3}{4})$ $\quad s = 1$ mile.

$39,092.50$

$C(\frac{3}{4}) = 5280 \cdot 5 \cdot \frac{5}{4} + 3(5280)\frac{1}{4}$

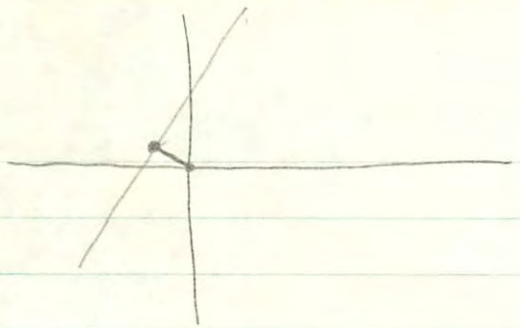
$34,980.$

if $s < \frac{3}{4}$ run line direct to factory if $s > \frac{3}{4}$ run line to a pt 3/4 mile downfrom E then along land to f.

this should be proven.

m is

(22) $y = 3x + 7$



$d^2 = (x-0)^2 + (y-0)^2$

$x^2 + (3x+7)^2$

$= x^2 + 9x^2 + 42x + 49$

$= 10x^2 + 42x + 49$

$(d^2)' = 20x + 42$

$x = -\frac{42}{20} = -\frac{21}{10}$

$y = -\frac{63}{10} + 7$

$y = \frac{7}{10}$

$\left(-\frac{21}{10}, \frac{7}{10}\right)$?

$- 0 +$ min f' test

$-\frac{21}{10}$ 0

General case? it kind of looks like $\left(\frac{-3m}{3+m}, \frac{3}{3+m}\right)$

$\frac{-ab}{a+b}, \frac{b}{a+b}$ close but ...

$y = mx + b$

$f = d^2 = x^2 + (mx+b)^2$

$= x^2 + m^2x^2 + 2mbx + b^2$

$f' = 2x + 2m^2x + 2mb$

$= 2[x(1+m^2) + mb] \overset{\text{set}}{=} 0$

$x = \frac{-mb}{1+m^2}$

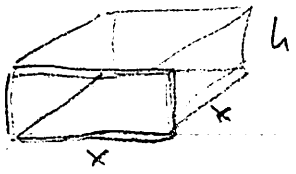
$y = m\left(\frac{-mb}{1+m^2}\right) + b$

$y = \frac{-m^2b + b + m^2b}{1+m^2}$

$= \frac{b}{1+m^2}$

generally! $\left(\frac{-mb}{1+m^2}, \frac{b}{1+m^2}\right)$

(27)



V fixed

$V = x^2 h$

$\frac{V}{x^2} = h$

$H = 4xh + 3x^2$

$H = \frac{4V}{x} + 3x^2$

$H' = \frac{-4V}{x^2} + 6x \overset{\text{set}}{=} 0$

$\frac{-4V + 6x^3}{x^2} = 0$

$6x^3 = 4V$

$x^3 = \frac{4V}{6}$

$x^3 = \frac{2}{3}V$

$x = \sqrt[3]{\frac{2V}{3}}$

$V = x^2 h$

$V = \left(\frac{2V}{3}\right)^{2/3} h$

$V^3 = \frac{4V^2}{9} h^3$

$V = \frac{4}{9} h^3$

$\frac{V}{x^2} = h$

$\frac{4}{9}\frac{h^3}{x^2} = h$

$\frac{4}{9} h^2 = x^2$

$\frac{2}{3} h = x$

$h = \frac{3}{2} x$

the bldg should be $\frac{3}{2}$ as high as it is wide

(33) $S = 42 - \frac{D}{3}$ D is Density ie # of veh/mile. S is Speed miles/hr.

a) we want veh/hr.

$\frac{\text{veh}}{\text{mile}} \cdot \frac{\text{mile}}{\text{hr}} = \text{veh/hr}$. So SD is # veh/hr.

$$SD = 42D - \frac{D^2}{3}$$

b) $f = 42D - \frac{D^2}{3}$ f = flow is cars/hr.

$f' = 42 - \frac{2}{3}D \overset{\text{set}}{=} 0$

$\frac{2}{3}D = 42 \quad \frac{3}{2}$

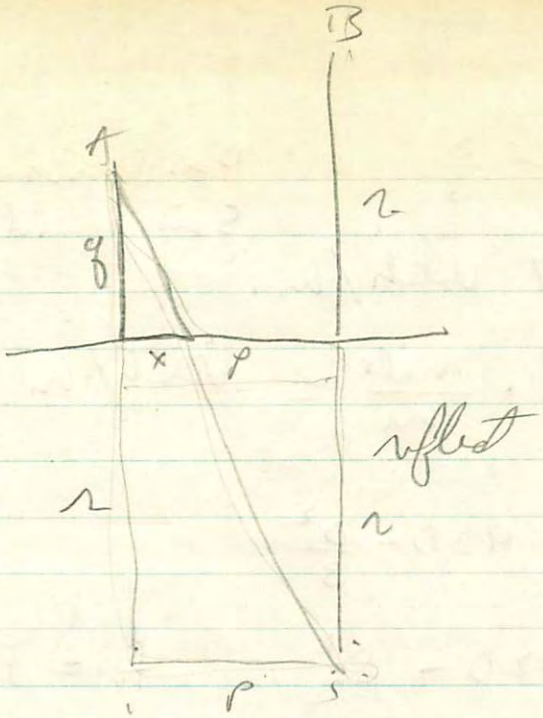
$D = 63$

when the density is 63 cars/mile the flow is at its max.

#36
Pg 226

Geo Version

Sim Δ ~ Lg Δ.

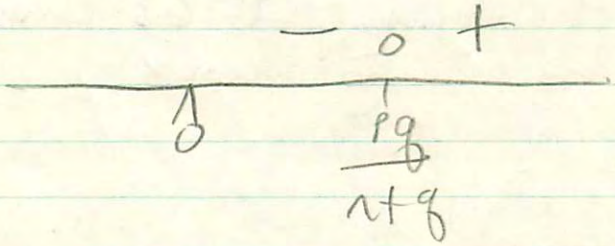


$$\frac{p}{q+R} = \frac{x}{q}$$

$$\frac{x}{q} = \frac{p}{r+q}$$

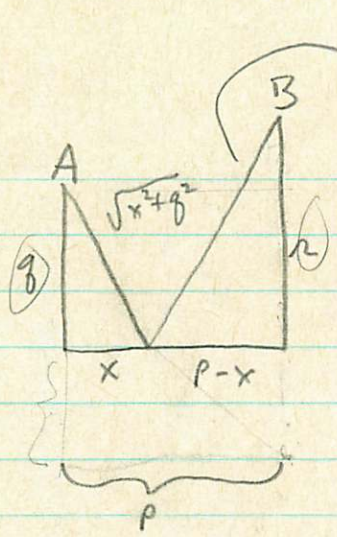
$$x = \frac{pq}{r+q}$$

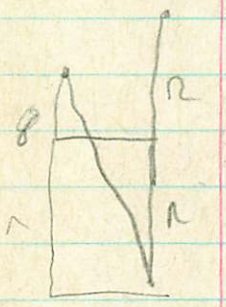
Verify



$f'(0) < 0$ min f'n test.

Pg 226





$f(x) = \sqrt{x^2+q^2} + \sqrt{r^2+(p-x)^2}$

$f'(x) = 2x + 2(p-x)(-1)$ this only if $q=r$

$= 2x - 2p + 2x$

$= 4x - 2p \overset{\text{set}}{=} 0$

$x = \frac{p}{2}$

$\sqrt{(q+r)^2+p^2}$

$f' = \frac{2x}{2\sqrt{x^2+q^2}} - \frac{2(p-x)}{2\sqrt{r^2+(p-x)^2}} \overset{\text{set}}{=} 0$

$x\sqrt{r^2+(p-x)^2} = (p-x)\sqrt{x^2+q^2}$

$x^2(r^2+p^2-2px+x^2) = (p^2-2px+x^2)(x^2+q^2)$

$x^2r^2 + x^2p^2 - 2px^3 + x^4 = p^2x^2 - 2px^3 + x^4 + p^2q^2 - 2pq^2x + q^2x^2$

quadratic in x

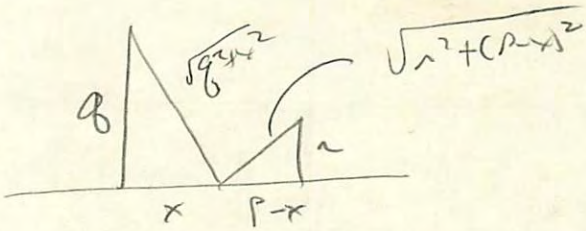
$(r^2-q^2)x^2 + 2pq^2x + p^2q^2 = 0$

$\frac{-2pq^2 \pm \sqrt{4p^2q^4 + 4p^2q^2r^2 - 4p^2q^4}}{2(r^2-q^2)}$ using "−" yields extraneous soln

$\frac{-2pq^2 + \sqrt{4p^2q^2r^2}}{2(r^2-q^2)}$

$\frac{-2pq^2 + 2pqR}{2(r^2-q^2)}$

$\frac{pq(R-q)}{(r+q)(r-q)}$ $\qquad \frac{pq}{r+q} = x$



$$f(x) = \sqrt{q^2+x^2} + \sqrt{\lambda^2+(P-x)^2}$$

$$f' = \frac{x}{\sqrt{q^2+x^2}} - \frac{(P-x)}{\sqrt{\lambda^2+(P-x)^2}} \stackrel{\text{set}}{=} 0$$

$$x\sqrt{\lambda^2+(P-x)^2} = (P-x)\sqrt{q^2+x^2}$$

$$x^2\left(\lambda^2+(P-x)^2\right) = \frac{(P^2-2Px+x^2)}{(P-x)^2}(q^2+x^2)$$

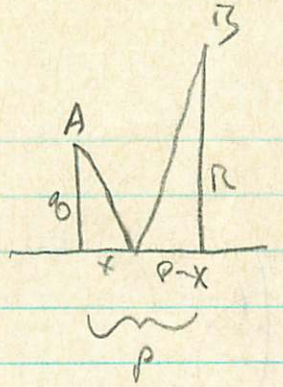
$$x^2\lambda^2 + x^2(P-x)^2 = q^2(P-x)^2 + x^2(P-x)^2$$

$$x^2\lambda^2 = q^2(P-x)^2$$

$$x\lambda = qP - qx$$

$$x(\lambda+q) = Pq$$

$$x = \frac{Pq}{\lambda+q}.$$



$$f(x) = \sqrt{q^2 + x^2} + \sqrt{R^2 + p^2 - 2px + x^2}$$

$$f'(x) = \frac{x}{\sqrt{q^2+x^2}} + \frac{x-p}{\sqrt{r^2+p^2-2px+x^2}}$$

x^2 $\qquad$ $p^2 - 2px + x^2$

$$x\sqrt{r^2+p^2-2px+x^2} = (p-x)\sqrt{q^2+x^2}$$

$$x^2r^2 + p^2x^2 - 2px^3 + x^4 = p^2q^2 - 2pq^2x + q^2x^2 + p^2x^2 - 2px^3 + x^4$$

$$x^2r^2 - q^2x^2 + 2pq^2x - p^2q^2 = 0$$

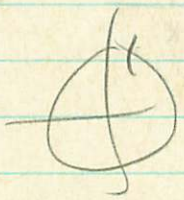
$\times$

$$(r^2 - q^2)x^2 + 2pq^2x - p^2q^2 = 0$$

$$x = \frac{-2pq^2 \pm \sqrt{4p^2q^4 + 4p^2q^2r^2 - 4p^2q^4}}{2(r^2+q^2)}$$

$$\frac{-2pq^2 \pm 2p^2qR}{r^2+q^2}$$

$f(\theta) = 27\csc\theta + 8\sec\theta$



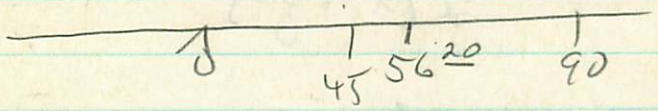
$f' = -27\csc\theta\cot\theta + 8\sec\theta\tan\theta$

$f'(45) = -27\sqrt{2}\cdot 1 + 8\sqrt{2}\cdot 1 < 0$

$f'(60) = -27\frac{2}{\sqrt{3}}\frac{1}{\sqrt{3}} + 8\cdot 2\sqrt{3}$

$-18 + 16\sqrt{3} > 0$

$- \quad 0 \quad +$



$f'(30)$.

so we have a min. by f' test

Pg 225
#32

PP. 225 #32

$1+2x^2$ $/TRUCKLOAD $\Rightarrow$ x = # OF TRUCKS (ROUTE #1)

$2+x^2$ $/TRUCKLOAD $\Rightarrow$ x = # OF TRUCKS (ROUTE #2)

Let t = # OF TRUCKS TO USE ROUTE #1
the cost to use that road is:
$(1+2t^2)t$ dollars

Since 1000 truckloads are needed to finish the job, the other route must handle:
$(1000-t)$ trucks

∴ the cost to use route # 2 is:
$(1000-t)[2+(1000-t)^2]$ dollars

the total cost of (both routes) is:
$C(t) = (1+2t^2)t + (1000-t)[2+(1000-t)^2]$

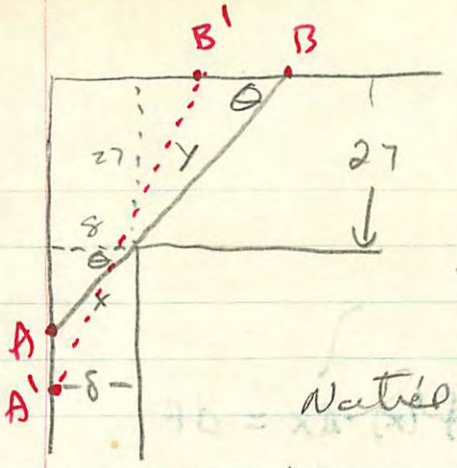
$C'(t) = [t(4t)+1+2t^2] + [[2+(1000-t)^2](-1)+(1000-t)[2(1000-t)(-1)]]$

$= 6t^2+1-2-(1000-t)^2-2(1000-t)^2$
$= 6t^2-1-3(1000-t)^2$
$= 6t^2-1-3(1000,000-2000t+t^2)$
$= 3t^2+6000t-3,000,001.$

setting C'(t) to 0 & using the quadratic formula
$t \doteq 414.2137$ or $t = -2414.2137$
since t cannot be negative, we disregard the 2nd choice. & since we cannot use .2137 of a truck we choose the closest whole value which is 414, or 415.
$C(414) = 3.431483 \times 10^8$
$C(415) = $ [illegible] $\times 10^8$ ∴ t = 414 is the best choice.
∴ 414 USE ROUTE # 1 & 586 USE ROUTE #2.

Scott Rath

35 $0 \le \theta \le 90$



Let $x+y$ be longest pipe which can be carried through hallway.

Notice $\csc\theta = \frac{Y}{27}$ $\sec\theta = \frac{X}{8}$

$\therefore$ $Y = 27\csc\theta$ $x = 8\sec\theta$

$\therefore$

$$\ell(\theta) = 27\csc\theta + 8\sec\theta$$

$$\ell'(\theta) = -27\csc\theta\cot\theta + 8\sec\theta\tan\theta$$

$$= -27\frac{\cos\theta}{\sin^2\theta} + 8\frac{\sin\theta}{\cos^2\theta} \overset{\text{set}}{=} 0$$

$$8\sin^3\theta - 27\cos^3\theta = 0 \Rightarrow \tan^3\theta = \frac{27}{8}$$

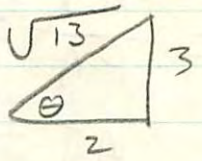
$$\tan\theta = \frac{3}{2}$$

$$(2\sin\theta - 3\cos\theta)(4\sin^2\theta + 6\sin\theta\cos\theta + 9\cos^2\theta) = 0$$

Not zero should be proved. if $\ell'(\theta) = 0$ & increases it's.

$\therefore$ $2\sin\theta = 3\cos\theta$

$$\tan\theta = \frac{3}{2} \Rightarrow$$



$\therefore$ $\sin\theta = \frac{3}{\sqrt{13}}$; $\cos\theta = \frac{2}{\sqrt{13}}$ $\theta \approx 56°20'$

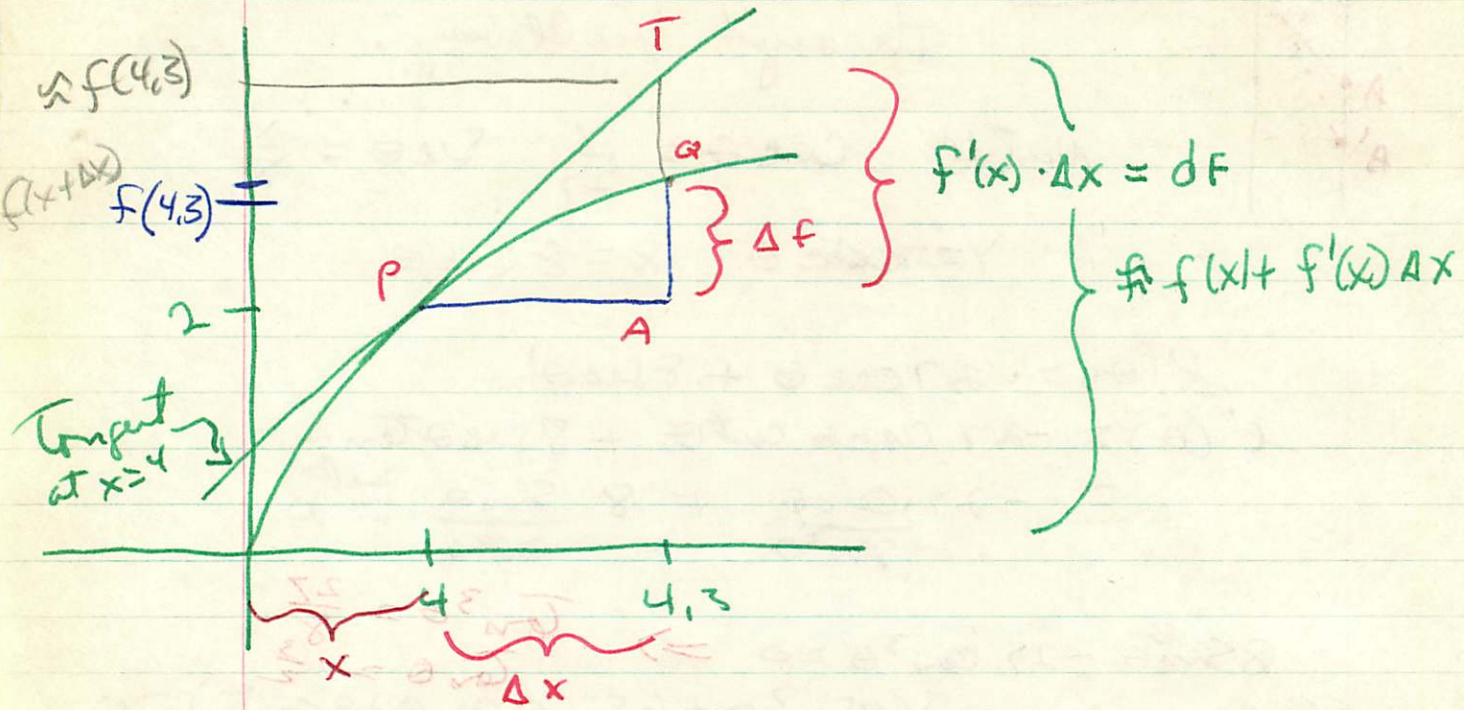
$\ell(56°20')$ = length or use expression above.

$$\ell(\theta) = 27\cdot\frac{\sqrt{13}}{3} + 8\cdot\frac{\sqrt{13}}{2}$$

$$9\sqrt{13} + 4\sqrt{13} = 13\sqrt{13} \approx 46.87$$

using ℓ' test check $\theta = 45°$, $\theta = 60°$

Notice: we actually want line segment $\overline{AB}$ to be a minimum! eg $A'B'$ is longer than $\overline{AB}$



$$\frac{\text{change } y}{\text{change } x} = \frac{AT}{PA} = f'(x)$$

$$AT = f'(x)\, PA$$

$$AT = f'(x)\, \Delta x \qquad PA = \Delta x$$

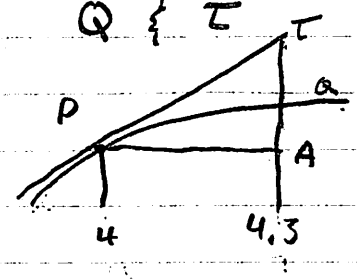
The differential

pg 226–232

6.9

eg 1 Let $f(x) = \sqrt{x}$ $x = 4$ $\Delta x = 0.3$

compute the difference in the y coord of Q & T



Q is actual value & T is approximated value of f(x) at $x = 4.3$

$$f(4.3) = f(4) + \overline{AQ} = f(4) + \Delta f$$

Consider the approximation at T:

$f(4) + \overline{AT}$ Also

$\frac{\overline{AT}}{\overline{PA}}$ = slope of tangent ie $f'(4)$ ∴,

$\overline{AT} = f'(4) \cdot \overline{PA}$ $\quad f'(x) = \frac{1}{2\sqrt{x}}$ so $f'(4) = \frac{1}{4}$

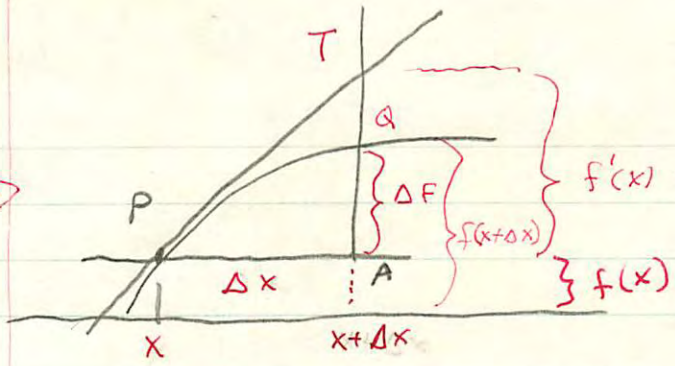
$f'(4)\ \Delta x$

$0.25 \cdot 0.3 = 0.075$ So Y coord at T is $f(4)$

$2 + 0.075 = 2.075$.

Y coord at Q is $\sqrt{4.3} \approx 2.0736$

So we have an error in the approximation of about -0.0014, ie very close approx.



Δx is change in x, Δf change in f. Since PT is tangent to f at P we say that the y coord of T is a good approx of f at (x+Δx). Providing Δx is small

$$f'(x) = \frac{\overline{AT}}{\overline{PA}} \Rightarrow \overline{AT} = \overline{PA} \cdot f'(x) = \Delta x \cdot f'(x)$$

$f'(x) \cdot \Delta x$ is called the **Differential** of f at x it is usually abbreviated **df**. Thus

$$f'(x) \cdot \Delta x = df$$

Approx ↓

eg 2 $f(x) = x^3$ $x = 5$ $\Delta x = 0.1$ Compute df Δf

↑ Actual

$$df = f'(x) \cdot \Delta x \Rightarrow 3x^2 \cdot 0.1 \Big\}_{x=5} = 7.5$$

See above Diagram →

$$\Delta f = f(5+\Delta x) - f(5)$$

$$(5.1)^3 - 5^3 = 132.651 - 125 = 7.651$$

So df & Δf are very close.

Furthermore from $f(x+\Delta x)$ can be approximated by $f(x) + f'(x)\Delta x$

$\Delta x = dx$

To clarify what it means to say that we have a good approximation when Δx is small is that df & Δf are the same when Δx is small. This can be stated:

WTP $\lim_{\Delta x \to 0} \frac{\Delta f}{df} = 1$

Consider: Note that $f'(x) \neq 0$

$$\frac{\Delta f}{df} = \frac{\Delta f}{f'(x) \cdot \Delta x} = \frac{\Delta f}{\Delta x} \cdot \frac{1}{f'(x)}$$

$\Delta f = f(x+\Delta x) - f(x)$ (From Picture)

$$\frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} \cdot \frac{1}{f'(x)}$$ — now take limit as $\Delta x \to 0$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \cdot \frac{1}{f'(x)}$$

$$= f'(x) \cdot \frac{1}{f'(x)} = 1$$

additional comments.

$$\frac{dy}{dx} = y'$$

~~What is dx? WRT x~~

this is called the differential of x

~~What dy? deriv of y?~~

Actually neither dy nor dx has had a def.

Their ratio is y'.

Now individually they have meaning.

dy is diff of y if $dy = y'\Delta x$

dx is diff of x if $\Delta x = dx$

$f(x) = x$

$df = f'(x)\,dx$

is $1\,dx$

Lecture 4.4
P. 229

Eg 3 Side of a cube is measured w/ an error of at most 1%. What % error may this cause in calculating the volume?

Let x = length of side V = volume
dx = error. $\therefore$ Relative error or Percent of error $\frac{dx}{x}$

So $\frac{|dx|}{x} \leq .01$

% error in volume.

dV = error in volume so $\frac{dV}{V}$ = Relative ERROR

$V = x^3$ Definition

$\frac{dV}{dx} = d(x^3) \Rightarrow \frac{dV}{dx} = 3x^2 \Rightarrow dV = 3x^2\,dx \Rightarrow$

$= 3x^2\,dx$ so $\frac{dV}{V} = \frac{3x^2\,dx}{x^3}$

$\frac{dV}{V} = \frac{3\,dx}{x} = 3\frac{dx}{x}$ so relative error in volume is 3 time the error of the side measurement.

Summary. $df = f'(x) \cdot \Delta x$ So

$d(\ln x) = \frac{1}{x}\Delta x \qquad d(e^x) = e^x \Delta x \qquad d(x) = 1 \cdot \Delta x$

Notice $dx = \Delta x$ so $df = f'(x)\,dx$ or if the fcn is described as y we have

$dy = y'\,dx \Rightarrow \frac{dy}{dx} = y'$

~~do pblm 23 for Summary.~~

(12) $d(e^{-x}) = -e^{-x}\,dx$

(13) $d(\ln(1+x)) = \frac{1}{1+x}\,dx$

(14) $d(\sqrt{1+x^2}) = \frac{2x\,dx}{2\sqrt{1+x^2}} = \frac{x\,dx}{\sqrt{1+x^2}}$

(15) $d(\tan^{-1}x) = \frac{dx}{1+x^2}$

(16) $d(\sec x^2) = 2x \sec x^2 \tan x^2\,dx$

(17) $d(\sin^{-1} 3x) = \frac{3\,dx}{\sqrt{1-9x^2}}$

(18) $d(\sec^{-1} 5x) = \frac{5\,dx}{|5x|\sqrt{25x^2-1}} = \frac{dx}{|x|\sqrt{25x^2-1}}$

(19) $d(2^{x^2})$

$2 = e^{\ln 2}$

$2^{x^2} = e^{x^2 \ln 2}$

$2x \ln 2\, e^{x^2 \ln 2}\,dx$

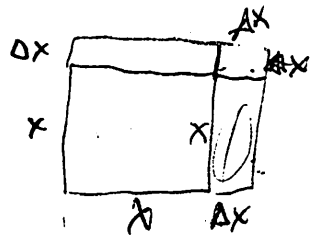
$2x \ln 2 \cdot 2^{x^2}\,dx$

(20) $d(\cos 5x) = -5 \sin 5x\,dx$

(21) $A = x^2$ $\qquad \frac{dx}{x} = .05$

$\frac{dA}{A} = \frac{2x\,dx}{x^2} = \frac{2\,dx}{x}$ $\qquad$ max $.1 = 10\%$

(23)



$f = x^2$

$\Delta f = (x+\Delta x)^2 - x^2$

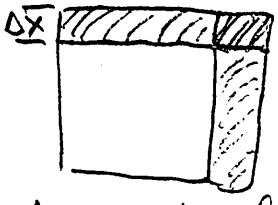
$= x^2 + 2x\Delta x + (\Delta x)^2 - x^2$

$= 2x\Delta x + (\Delta x)^2$

$df = f'(x)\,\Delta x$

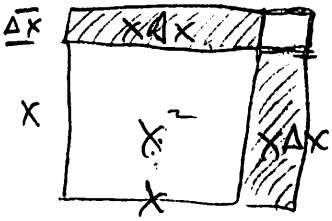
$2x\,\Delta x$ is the difference is $(\Delta x)^2$

b)



Δf shaded.
Actual change

c)

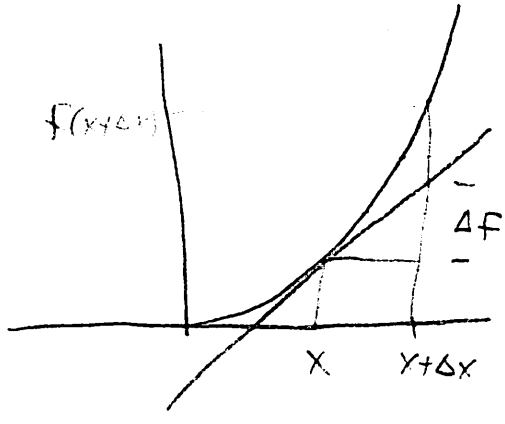


df shaded
Approximate change.

(25) another version of chain rule

$f(x+\Delta x) \approx f(x) + f'(x)\,\Delta x$

(23) $y = x^2$



Δf

Δf

$\Delta f \quad (x+\Delta x)^2 = x^2 + 2x\Delta x + (\Delta x)^2$

$x\Delta x + x\Delta x$

$f(x) = x^2$

$\frac{df(x)}{dx} = 2x$

Δx

$\frac{df}{dx} = 2x$

GQ Chapt 6 pg 242

9. $A = \pi r^2$

a) $dA = 2\pi r\, dr$. b) $2\pi r$ is the derivative of πr^2

(18) $\sqrt{25+dx} \approx 5 + \frac{dx}{10}$

a)

$$\lim_{dx\to 0} \frac{\sqrt{25+dx}}{5+\frac{dx}{10}} = \lim_{dx\to 0} \frac{\sqrt{25+dx}}{\frac{50+dx}{10}} = \lim_{dx\to 0} 10\,\frac{\sqrt{25+dx}}{50+dx}$$

$\frac{10\cdot 5}{50} = 1$ ∴ good approx.

b) $\sqrt{a^2+dx} \approx a + \frac{dx}{2a}$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$\lim_{dx\to 0} \frac{\sqrt{a^2+dx}}{a+\frac{dx}{2a}} = \lim_{dx\to 0} \frac{\sqrt{a^2+dx}}{\frac{2a^2+dx}{2a}} = \lim_{dx\to 0} \frac{2a\sqrt{a^2+dx}}{2a^2+dx}$$

$\frac{2a^2}{2a^2} = 1$

$\sqrt{a^2+dx} \approx \sqrt{a^2} + \frac{1}{2\sqrt{a^2}}\cdot dx$

$a + \frac{dx}{2a}$

c) $\sqrt{65.6}$ $\quad 8^2 + 1.6$ $\quad \sqrt{a^2+dx} \approx a + \frac{dx}{2a}$

$\sqrt{65.6} = 8 + \frac{1.6}{16} = 8 + .1 = 8.1$

$\sqrt{67}$ $\quad 64+3$ $\quad 8.\frac{3}{16}$

(10) $\frac{ds}{s} = .02$ $\quad A = 6s^2$

$$\frac{dA}{A} = \frac{12s\,ds}{6s^2}$$

$$= \frac{2ds}{s} = 2(.02) = 4\% \text{ error}$$

we want $\frac{df}{f}$ where f is the surface area fctn.

Surface area of the cube is $6 \cdot x^2 = 6x^2$.

we want to estimate $\frac{\Delta f}{f}$

$\Delta f \approx df = f'(x)\,dx$

We know the error in the measurement of the side is 2% that is $dx = (.02)x$

ie $\frac{dx}{x} = .02$, so dx is the error.

Now, $f(x) = 6x^2$ (fctn of terms of the side) (Surface Area)

So $f'(x) = 12x$

$\Delta f \approx df = 12x(.02)x = .24x^2$

So $\frac{df}{f} = \frac{.24x^2}{6x^2} = .04$

error is about 4%

7 a) $\sqrt[3]{8+h} \approx 2 + \frac{h}{12}$

$\lim_{h \to 0} \frac{\sqrt[3]{8+h}}{2 + \frac{h}{12}} = 1$

$\frac{\text{Actual} - \text{ESTimate}}{\text{ACTual}} = \%$

b) $h = 1 \quad \sqrt[3]{9} \quad 2 + \frac{1}{12}$

2,0801 2,0833

2,0833
2,0801
.0032 error.

using $\square .\% = .0032$

$\% \text{ error} = \frac{.0032}{2,0801}$

$\frac{32}{20801} = .15\%$ error.

$h = -1 \quad \sqrt[3]{7} \approx 2 - \frac{1}{12}$

$1.9129 - 1.9167 = .0038$

$\frac{38}{19129} \approx .2\%$ error

(8) $\frac{dy}{dx} = -y^2 \qquad y = x^a$

$\frac{dy}{y^2} = -dx \qquad \frac{dy}{dx} = ax^{a-1} = -x^{2a}$

$a = -x^{2a-a+1}$

$a = -x^{a+1}$

(27) $u = x^5 \quad du = 5x^4\,dx$

(28) $u = \tan^{-1} 3x \quad du = \dfrac{3\,dx}{9x^2+1}$

(29) $u = -\cos 5x \quad du = 5\sin 5x\,dx$

(35) $\tan^{-1}(1+h) \approx \dfrac{\pi}{4} + \dfrac{h}{2}$

Let $f(x) = \tan^{-1} x \quad dx = h \quad x = 1$

$f'(x) = \dfrac{1}{1+x^2}$

$f(x+h) \approx f(x) + f'(x)\,dx$

$\tan^{-1}(1+h) \approx \tan^{-1} 1 + \dfrac{1}{1+1^2} \cdot h$

$\dfrac{\pi}{4} + \dfrac{h}{2}$

a) $\lim_{h \to 0} \dfrac{\tan^{-1}(1+h)}{\frac{\pi}{4} + \frac{h}{2}} =$

b) $\tan^{-1} 0.8 \quad h = -.2$

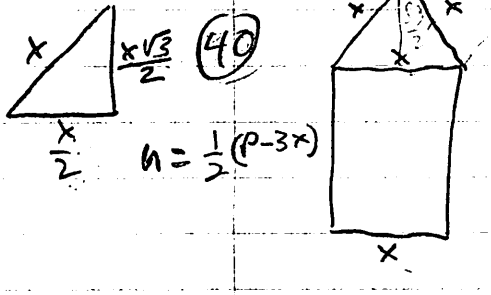
$\dfrac{3.14}{4} - \dfrac{.2}{2} = \dfrac{3.14}{4} - .1 = \dfrac{3.14 - .4}{4} =$

$\dfrac{2.74}{4} \approx 4.69\%$

c) $.69$ $\quad \tan^{-1} .8 = .675 \quad \left(\dfrac{\text{Act} - \text{Est}}{\text{Act}} = \%\right)$

mean $\dfrac{.015}{.675}$ 2.22%

(40)



Let P = Perimeter (constant) want to max area.

$A(x) = \dfrac{x}{2}(P - 3x) + \dfrac{x^2\sqrt{3}}{4} = \dfrac{1}{4}(2Px - 6x^2 + x^2\sqrt{3})$

$A'(x) = \dfrac{1}{4}(2P - 12x + 2x\sqrt{3}) \overset{\text{Set}}{=} 0$ Notice $f'' < 0$ ∴ we have max

$2P = 12x - 2x\sqrt{3}$

$2P = (12 - 2\sqrt{3})x$

$\dfrac{P}{(6-\sqrt{3})} = x$

$h = \dfrac{1}{2}\left(P - \dfrac{3P}{6-\sqrt{3}}\right)$

$= \dfrac{1}{2}\left(\dfrac{6P - P\sqrt{3} - 3P}{6-\sqrt{3}}\right)$

$= \dfrac{3P - P\sqrt{3}}{2(6-\sqrt{3})} = \dfrac{(3-\sqrt{3})P}{2(6-\sqrt{3})} = h$

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Define $f(x) = x^N - e^x$ $\quad N \geq 3$

$f(0) = 0 - 1 = -1$

$f(N) = N^N - e^N > 0$ since $N \geq 3$ and $3 > e$

$\lim_{x \to \infty} f(x) = -\infty$ because the exponential e^x gets bigger than the monomial x^N.

So, first negative,

IT become positive crossing x axis

IT again becomes negative crossing again.

Let Y = # of People

(45) $\frac{dY}{dt} = 21 - 8 = 13$ mill

$Y = Ae^{kT}$

$Y = 570e^{kT}$

$1000 = 570e^{25k}$

$\left(\frac{1000}{570}\right)^{\frac{1}{25}} = e^k$ $\quad k = \frac{1}{25}\ln\left(\frac{100}{57}\right)$

$1.0227 = e^k$ $\quad k = .0225$ Actual

$1 + .0227 = e^k \approx 1 + k$ Compound.

approx $\hat{=}$ value of k

a) ≈ 2000 1 Billion. Given.

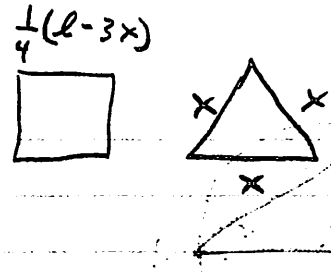
b) INSTANEOUS growth rate $k = .0225$

c) Percent growth per year.

$$\frac{\text{Population 1 year later} - \text{Start Population}}{\text{start Population.}}$$

$$\frac{570 \cdot e^k - 570}{570} = e^k - 1 = .0227 \text{ or } 2.27\% \text{ growth.}$$

(41) l = length of wire



a) sum of area to be min.

$$A(x) = \left(\frac{l-3x}{4}\right)^2 + \frac{x^2\sqrt{3}}{4}$$

$$= \frac{l^2 - 6lx + 9x^2}{16} + \frac{4x^2\sqrt{3}}{16}$$

$$= \frac{1}{16}\left(l^2 - 6lx + 9x^2 + 4x^2\sqrt{3}\right)$$

$$A'(x) = \frac{1}{16}\left(-6l + 18x + 8x\sqrt{3}\right) \overset{\text{set}}{=} 0$$

$$3l = 9x + 4x\sqrt{3}$$

$$\frac{3l}{9+4\sqrt{3}} = x \quad \text{Side of } \Delta$$

$\therefore$ cut wire at $\frac{9l}{9+4\sqrt{3}}$ (ie we need 3 sides)

$$\frac{1}{4}\left(l - \frac{9l}{9+4\sqrt{3}}\right)$$

$$\frac{1}{4}\left(\frac{9l+4l\sqrt{3}-9l}{9+4\sqrt{3}}\right) = \frac{l\sqrt{3}}{9+4\sqrt{3}}$$

Book answer disagrees ←

$$A'' = \frac{1}{16}\left(18+8\sqrt{3}\right) > 0$$

$\therefore$ min

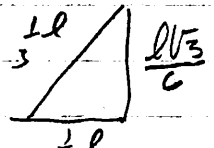
b) Same problem now looking for max, it must be at end pts, ie all wire used for □ or Δ.

$$A(\text{Square}) = \frac{l^2}{16} = .1 \qquad A(\text{Triangle}) = \frac{1}{6}l \cdot \frac{l\sqrt{3}}{6} = \frac{l^2\sqrt{3}}{36}$$

.0625 .0481

$$\frac{l^2}{16} > \frac{l^2\sqrt{3}}{36} \quad \therefore$$

all wire must be used for Square.



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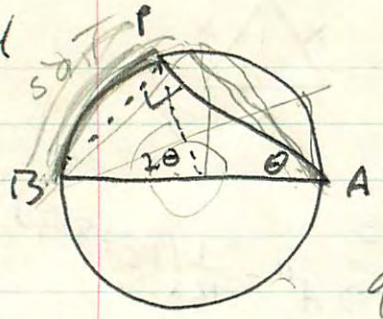


$\frac{\overline{AP}}{200} = \cos\theta$

$\overline{AP} = 200\cos\theta$

$\overset{\frown}{BP} = 2\theta \cdot \frac{\pi}{180}$

$AP = 200\cos\theta$

$0 \le \theta < 90$

$RT = D \therefore T = \frac{D}{R}$

$f(\theta) = \frac{2\theta}{200} + \frac{200\cos\theta}{100}$

$f(\theta) = \frac{100\theta}{100} + \frac{2\cos\theta}{1}$

$f(\theta) = \theta + 2\cos\theta$

$f'(\theta) = \frac{1}{100} - 2\sin\theta \overset{\text{set}}{=} 0$

$\frac{1}{2} \cdot \frac{1}{100} \quad .5 \cdot 0\%$

$\sin\theta = \frac{1}{200} = .005$

2π

$2\theta \cdot 100$

200θ

$2\theta = 5\% \pi$

$\theta = 45$

50π

50°

$\frac{\pi x}{180} = \frac{20\pi}{6}$

$x = 30.$

$f(\theta) = \theta + 2\cos\theta$

$f(x) = \frac{\pi x}{180} + 2\cos\left(\frac{\pi x}{180}\right)$

$f'(x) = \frac{\pi}{180} - 2\sin\left(\frac{\pi x}{180}\right)\frac{\pi}{180} \overset{\text{set}}{=} 0$

$\sin\left(\frac{\pi x}{180}\right) = \frac{1}{2}$

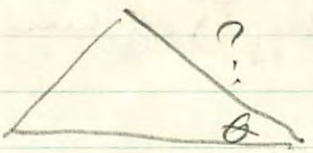
2πr

20π

2θ

$360 \cdot \frac{2\theta}{360}$

$90° \cdot 100 \cdot \frac{2\theta}{360}$



200

$\frac{?}{200} = \cos\theta$

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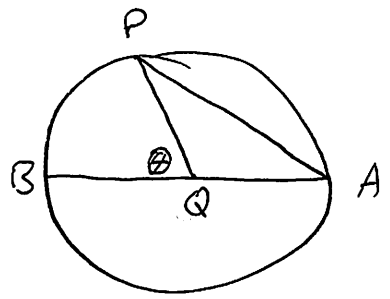
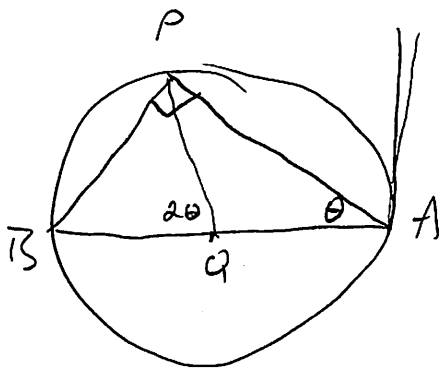


$q^2 = QA^2 + QP^2 - 2 QA \cdot QP \cos Q$

$q^2 = 100^2 + 100^2 - 2\,100^2 \cos Q$



$\frac{AP}{200} = \sin B = \sin\left(\frac{\pi}{2} - \theta\right)$

$= \sin\frac{\pi}{2}\cos\theta - \sin\theta\cos\frac{\pi}{2}$

$= 1$

$\frac{AP}{200} = \cos\theta \qquad 0 \le \theta < 90$

$AP = 200\cos\theta$

want this

$\overset{\frown}{BP} = r(2\theta)$

200θ

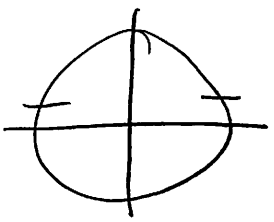
$f(\theta) = \frac{200\theta}{200} + \frac{200\cos\theta}{100}$

$f(\theta) = \theta + 2\cos\theta$

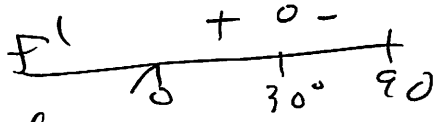
$f'(\theta) = 1 - 2\sin\theta \overset{\text{set}}{=} 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ i.e. 30°



$f\left(\frac{\pi}{4}\right) = 1 - \sqrt{2} < 0$



so we have max at 30°

but we require min!

$F(0) = 2$

$f(90) = 90$

Run around

$\frac{1}{2}C = \frac{1}{2} 2\pi r$

πr

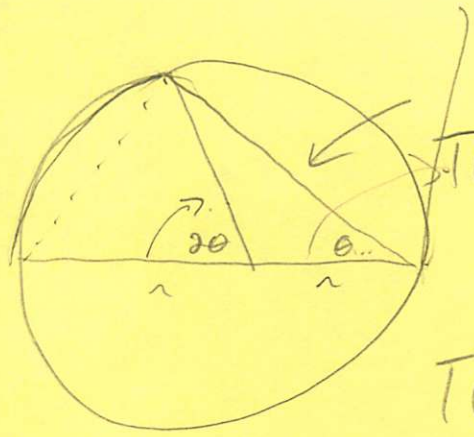
100π = dist around.

$\frac{100\pi}{200}$ = Time $\frac{\pi}{2}$ minutes. Swim across $\frac{200\text{ ft}}{100\text{ ft/sec}} = 2$ sec.

min when running around. $\approx 1\frac{1}{2}$ minutes.

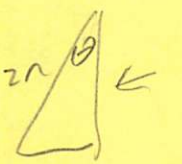
#64
Pg 247

$0 \le \theta \le 90$



$T(\theta) = \frac{1}{\frac{200}{100}} 2\theta \cdot \frac{\pi}{180} r + \frac{2r}{100} \cos\frac{\theta\pi}{180}$

$T(\theta) = \frac{r}{100}\left[\frac{\theta\pi}{180} + 2\cos\left(\frac{\theta\pi}{180}\right)\right]$



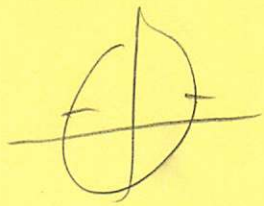
$T' = \frac{\pi}{180} - 2\sin\left(\frac{\theta\pi}{180}\right)\cdot\frac{\pi}{180}$

$\frac{\pi}{180}\left[1 - 2\sin\left(\frac{\theta\pi}{180}\right)\right] = 0$

$\sin\frac{\theta\pi}{180} = \frac{1}{2}$

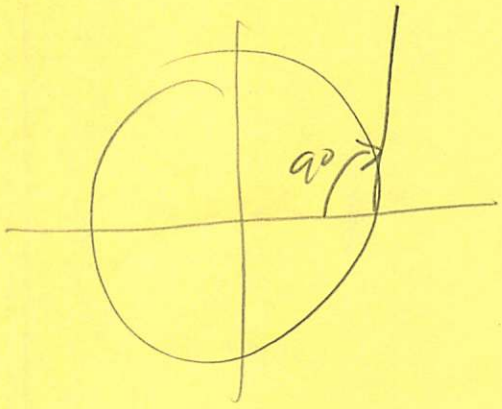
$T(0) = 2$

$T(90) = \frac{\pi}{2}$



$\frac{\theta\pi}{180} = \frac{\pi}{6}$

$\theta = 30^\circ$



$T'' = \frac{\pi}{180}\left(-2\cdot\frac{\pi}{180}\cos\frac{\theta\pi}{180}\right) < 0$

$\therefore$ max.

we want min so we check end points.

Since $\frac{\pi}{2} < 2$

he actually runs around.

(56) $f(-x) = f(x) \Rightarrow$ even

$f(-x) = -f(x) \Rightarrow$ odd

a) x^4 even

b) x^5 odd

c) $x^3 + x^2$ neither

d) $|x|$ even

e) $\tan x$ odd

f) $\sin 3x$ odd

g) e^x neither

h) e^{x^2} even.

(57) a) Curve (x, y) even then $(-x, y)$

b) " (x, y) odd then $(-x, -y)$

c) reflect through y axis or through origin

(58)

Aim. $g(x) = f(x) + f(-x)$

$g(-x) = f(-x) + f(-(-x)) = g(x)$ $\therefore$ g is even.

$h(x) = f(x) - f(-x)$

$h(-x) = f(-x) - f(-(-x))$

$= f(-x) - f(x)$

$= -h(x)$ $\therefore$ h is odd.

$h + g = f(x) + f(-x) + f(x) - f(-x)$

$h + g = 2f(x)$

Add together

Now solve for $f(x)$.

(70) WTP $a < \frac{a-b}{\ln a - \ln b} < b$ assume $a < b$

$a\ln a - a\ln b > a - b > b\ln a - b\ln b$

$(a-b)\ln a > (a-b) - b\ln a + a\ln b > (a-b)\ln b$

$\ln a < 1 < \frac{b\ln a + a - b\ln b}{(a-b)} < \ln b$

$a < b$

$\ln a < \ln b$

$\ln a - \ln b < 0$

$a < b$

$a - b < 0$

$a < \frac{a-b}{\ln a - \ln b}$

$a\ln a - a\ln b < a - b$

$\ln a - \ln b < 1 - \frac{b}{a}$

$a < b$

$a + a < a + b$

$a\ln a < b\ln a$

$(a-b)\ln a < 0 < a$

$b\ln b < b\ln b$

$(a-b)\ln b < 0 < b$

$(a-b)\ln a - (a-b)\ln b < a - b$

$(a-b)(\ln a - \ln b) < a - b$

$(a-b) < \frac{a-b}{\ln a - \ln b}$

Pg 248
#78

$$f(x) = \left(\frac{e^x+1}{2}\right)^{\frac{1}{x}}$$

a) $\lim_{x\to 0^+} f(x)$

let $y = f(x)$

$$\ln y = \frac{1}{x}\ln\left(\frac{e^x+1}{2}\right)$$

$$= \frac{1}{x}\left[\ln(e^x+1) - \ln 2\right]$$

b)

$$\lim_{x\to\infty} \frac{\ln(e^x+1) - \ln 2}{x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to 0^+}$$

$$= \frac{\ln(e^x+1) - \ln 2}{x} \overset{\frac{0}{0}}{=}$$

$$\lim_{x\to\infty} \frac{e^x}{e^x+1} \overset{\frac{\infty}{\infty}}{=}$$

$$\lim_{x\to 0^+} \frac{e^x}{e^x+1} = \frac{1}{2}$$

$$\lim_{x\to\infty} \frac{e^x}{e^x}$$

$$\ln y = \frac{1}{2}$$

$$y = e^{\frac{1}{2}} = \sqrt{e}$$

$$\lim_{x\to\infty} 1$$

c) WTS $\ln f$ increases

$$\ln y = 1$$

$$y = e^1 = e$$

85b $f(x) = \frac{b}{a}\sqrt{x^2-a^2}$

$f(a) = f(-a) = 0$

$f'(x) = \frac{bx}{a\sqrt{x^2-a^2}}$ u v

$f''(x) = ab\sqrt{x^2-a^2}$ -

$= ab\left(\frac{x^2-a^2-x^2}{\sqrt{x^2-a^2}}\right) = ab\left(\frac{-1}{\sqrt{x^2-a^2}}\right)$

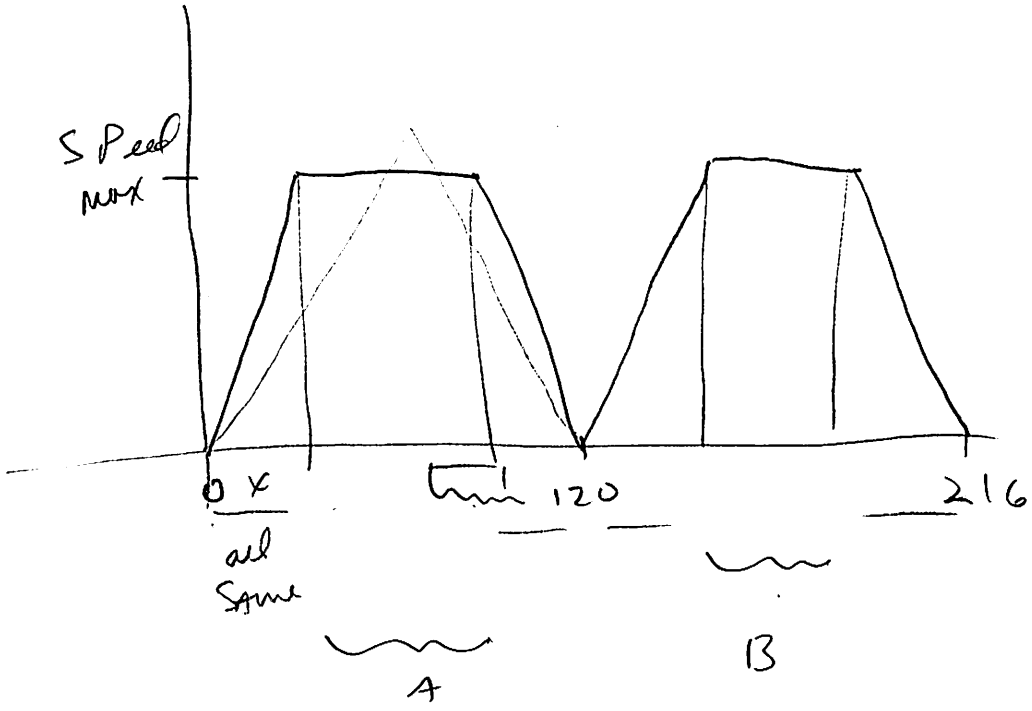
$= \frac{-ab}{\sqrt{x^2-a^2}}$

f is economy

f is a decreasing factor which is Nathan steeps as it ~~but f'' is just > 0 so~~ uses.

120 sec 30 blocks

96 sec 24 blocks



$A+B+4x = 216$

$A+2x = 120$

$A-B = 24$.

$B+2x = 96$

Calc C 6 app

The Continuing Legend of Bagger Vance

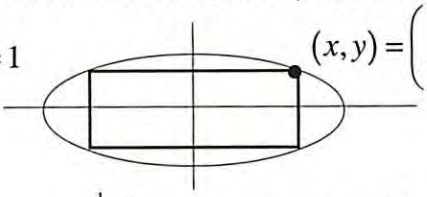
Mr. Mumaugh

Bagger Vance was first seen in the company of Captain Runnulph Junuh and Harley Greaves. Adele Invergordon provided the setting in Savannah, Georgia in the 1930s and you know the story .....
His legend now continues since Bagger has reappeared in Vacaville. He has seen your calculus work and in his soft spoken style he said "Some things must be learned so they can be remembered." "The rhythm of calculus is just like the rhythm of life." "Find *your* place in the field. Focus. Be there!" "Identify your variable then translate the given into a function." "Draw a useful, labeled picture." "Write down what you *do* know and what you *do not* know. Make the two into one." "Always include justification." "Write a brief explanation of your method." "Convince the reader that *you* know the concept." "It is the closest point because *you* said it was." "When we question the path we've taken, the answer can be right in front of our eyes." "Use a caclulator when necessary. Remember to check your work with a calculator." "3 places after the point accuracy in all decimal answers is expected." "Enjoy."

Bagger sees that you are faced with 18 of the most demanding holes you've ever encountered. He will be there to help but you must remember that "It is your game, not Bagger's," so *you* must rise to the occasion.

1. Given : $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$ for $-1 \le x \le 4$. Find all significant points. Sketch graph. Verify your work with your grapher.
2. The volume of 1 kg of water at a temperature T between $0°$ C and $30°$ C is approximately $V = 999.87 - (0.06426)T + (0.0085043)T^2 - (0.0000679)T^3$ cubic centimeters. At what temperature does water have its maximum density?
3. What is the maximum possible area of a rectangle with base that lies on the x-axis and with two upper vertices that lie on the graph of the equation $y = 4 - x^2$?
4. A printing company has eight presses, each of which can print 3600 copies per hour. It costs \$5.00 to set up each press for a run, and $10 + 6n$ dollars to run n presses for 1 hour. How many presses should be used in order to print 50,000 copies of a poster most profitably?
5. A farmer wants to hire workers to pick 900 bushels of beans. Each worker can pick 4 bu/hr and is paid \$1.00/bu. The farmer must also pay a supervisor \$10 per hour while the picking is in progress, and he has miscellaneous additional expenses of \$8 for each worker hired. How many workers should he hire in order to minimize the total cost? What will then be the cost per bushel picked?
6. The heating and cooling costs for a certain uninsulated house are \$500 per year, but with $x \le 10$ in. of insulation the costs would be $\frac{1000}{2+x}$ dollars per year. It will costs \$150 for each inch (thickness) of insulation installed. How many inches of insulation should be installed in order to minimize the *total* (initial plus annual) costs over a 10-year period? What will then be the annual savings resulting from this optimal insulation?
7. A concessionaire had been selling 5000 wieners each game night at 50¢ each. When she raised the price to 70¢ each, sales dropped to 4000 per night. If she has fixed costs of \$1000 per night and each wiener costs her 25¢, what price will maximize her nightly profit?

8. A commuter train carries 640 passengers each day from a suburb to a city. It costs \$1.50 per person to ride the train. It is found that 40 fewer people will ride the train for each 5¢ increase in the fare. What fare should be charged to make the largest possible revenue?
9. Show that the rectangle with maximum perimeter that can be inscribed in a circle is a square.
10. Find the dimensions of the rectangle (with vertical and horizontal sides) of maximum area that can be inscribed in the ellipse with equation $\frac{x^2}{25}+\frac{y^2}{9}=1$



$(x,y)=\left(x,\frac{\sqrt{225-9x^2}}{5}\right)$

11. There is exactly one point on the graph of $y=(3x-4)^{\frac{1}{3}}$ closest to the origin. Find it.
12. A toy manufacturer is planning on mass-producing a new addition to their line of stuffed animals–an eagle. An efficiency expert estimates that if the price per eagle is x dollars, the profit will be given by the formula: $P=K\left(\frac{x-9}{x^2}-0.01\right)$ where $K>0$ for $x>0$. What price maximizes the profit?
13. Find the coordinates of the points on the graph of $f(x)=x^2-1$ which are nearest the origin.
14. Two long, straight railroad tracks cross each other at right angles. Train A, traveling at a constant velocity of 75 mph, crosses the intersection at 5 p.m. southbound on the North-South tracks. Train B, traveling east bound on the East-West tracks at 60 mph, crosses the intersection 10 minutes later. To the nearest second, at what time are the trains closest together?
15. A country whose only export product is coffee can sell x tons of coffee per month on the international market at the price of $\left(300-\frac{x}{1000}\right)$ dollars per ton. The cost of shipping x tons is 10 dollars per ton plus 1000 dollars overhead. Assuming the country has nationalized its coffee industry, what level of export will maximize the dollar income of the country?
16. A rectangle is inscribed in a right triangle, so that ~~one~~ Two sides of the rectangle lies on ~~one~~ the legs side of the triangle. Show that the area of the rectangle cannot exceed half the area of the triangle. (hint: *hyp* $y=mx+b$)
17. A farmer has 2040 feet of fencing and wishes to fence off two separate fields. One field is to be a rectangle with the length twice the width, while the other field is to be square. What do you advise the farmer to do so he can enclose the maximum area?
18. A company transports goods from Alamo City to Baker City. Materials are carried in trucks that travel at a speed of x mph. Legal speeds on the road are between 35 and 55 mph. Assume the fuel costs \$1.25 per gal per hour and is consumed at $4+x^2/200$. The Company pays its drivers \$11 per hour and reimburses them for the cost of fuel as well. You are the efficiency expert. What speed do you recommend the drivers use to minimize the cost per mile?

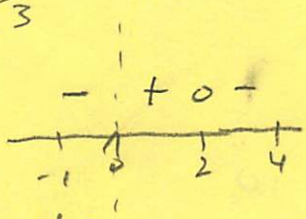
Bagger knows when it is time to move on. He has seen you prove yourself a worthy competitor and he is very proud of your accomplishments. Alas, even Bagger Vance doesn't know why you get honors credit for a class which is this easy!
Wave to him as he walks into the sunset.
.... knowing full well that you are a better calculus student as a direct result of knowing Bagger Vance and allowing his legend to continue.

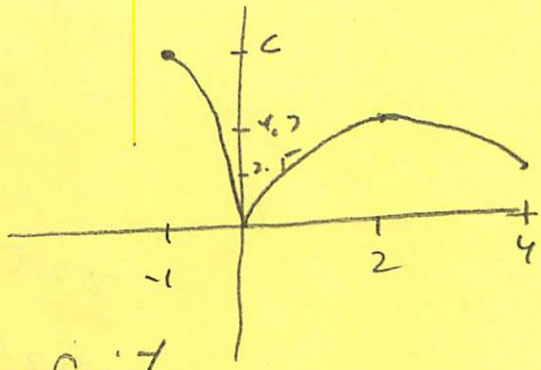
$x^{\frac{2}{3}}(5-x)$

(1) $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}} \quad -1 \le x \le 4$

$f'(x) = \frac{10}{3}x^{-\frac{1}{3}} - \frac{5}{3}x^{\frac{2}{3}}$

$= \frac{5}{3}\left(\frac{2-x}{x^{\frac{1}{3}}}\right)$





Abs: max END PT dec fnct.

$f(-1) = -5^{\frac{2}{3}} + 1 = 6$

$f(2) = 3\sqrt[3]{4} \approx 4.762$ max f' ust

$f(4) = \sqrt[3]{16} \approx 2.520$ min END PT dec fnct.

(0,0) Abs min dec fnct, inc fnct f' ONE

(2) $V = 999.87 - .06426T + .0085043T^2 - .0000679T^3$

max density at min volume :

$fmin(Y_1, X, 0, 30) = 3.967$

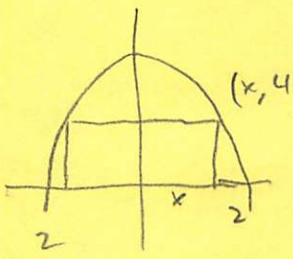
and that value is 999.745

$f(0) = 999.87$

$f(30) = 1003.763$

Absolutely no use not help on the grapher! Numbers too ugly for any other approach.

(3)



$A = lw$

$A(x) = 2x(4-x^2)$

$= 8x - 2x^3$

$A'(x) = 8 - 6x^2 \overset{set}{=} 0$

$x^2 = \frac{4}{3}$

$x = \frac{2\sqrt{3}}{3} \approx 1.155$

$A''(x) = -12x < 0 \ \forall x > 0$!

max f'' ust,

$A\left(\frac{2\sqrt{3}}{3}\right) = \frac{4\sqrt{3}}{3}\left(4 - \frac{4}{3}\right)$

$\frac{4\sqrt{3}}{3}\cdot\frac{8}{3} = \frac{32\sqrt{3}}{9} \approx 6.158$

max area enclosed is $\frac{32\sqrt{3}}{9}$

$35^2 = 1225$

2250

225 . 10

15 . 15 . 10

④ Let N = # of presses used. $\frac{50000}{3600N} = \frac{13.889}{N}$ hrs needed.

$f(N) = 5N + \left(\frac{10+6N}{\text{hour}}\right)\frac{50000}{3600}$ $\left(\frac{125}{9N}\right)$

$f(N) = 5N + \frac{125}{9N}(10+6N)$

$f(N) = 5N + \frac{1250}{9N} + \frac{250}{3}$

$f'(N) = 5 - \frac{1250}{9N^2} \overset{\text{Set}}{=} 0$

$45N^2 = 1250$

$N^2 = \frac{1250}{45}$

$= \frac{250}{9}$

$N = \frac{5\sqrt{10}}{3} = 5.270$

$f'' = \frac{2500}{9x^3} > 0$

$\therefore$ min at all $x > 0$.

since the choice is 5 or 6

$f(5) = 136.11$

$f(6) = 136.48$

$\therefore$ the company need to use 5 machines.

⑤ Let x = # of workers hired. $\frac{900}{4x}$ hrs required.

900 because workers are paid $4/hr.

$x \cdot \frac{900}{4x} \cdot 4 + 10 \cdot \frac{900}{4x} + 8x = f(x)$

(# of workers · # of hrs · $ per hr) — Amt Paid to workers; (Supervisor Pay · # hrs); misc exp

$900 + \frac{2250}{x} + 8x = f(x)$

$f' = -\frac{2250}{x^2} + 8 \overset{\text{Set}}{=} 0$

$8x^2 = 2250$

$x^2 = \frac{2250}{8}$

$x = \frac{15\sqrt{10}}{2\sqrt{2}}$

$\frac{15\sqrt{5}}{2}$

$f(16) = 1168.625$

$f(17) = 1168.353$

$f'' = +\frac{4500}{x^3} > 0 \; \forall x$ $\therefore$ 1 min

f'' test

The farmer needs 17 workers,

$\frac{1168.353}{900}$

about $1.30 per bushel

6. $f(x) = \left(\frac{1000}{2+x}\right).10 + 150x$

- $\frac{1000}{2+x}$: Per tree cost
- $.10$: for 10 years
- 150: $ per inch
- x: # of inches

$f(x) = \frac{10000}{2+x} + 150x$ 25.6

$f'(x) = \frac{-10000}{(2+x)^2} + 150 \overset{SET}{=} 0$

$10000 = 150(2+x)^2$

$100 = 5\sqrt{6}(2+x)$

$\frac{100}{2+x} = 5\sqrt{6}$

$\frac{100}{5\sqrt{6}} - 2 = x$

$\frac{20}{\sqrt{6}}$

$\frac{20\sqrt{6}}{6} - 2$

$\frac{10\sqrt{6}}{3} - 2 = x \approx 6.165$

$\frac{100}{5\sqrt{6}}$

$\frac{20}{\sqrt{6}}$

$\frac{20\sqrt{6}}{6}$

$\frac{10\sqrt{6}}{3} - 2 = x$

$f'' = \frac{20000}{(2+x)^3} > 0$ ∀x., min. $f''(6.165)$.

(7) Let x = increase in price

$50+x$ = New cost

$5000 - 50x$ = New # sold.

20¢ → 1000 Loss

20 1¢ → 50 Loss

$f(x) = (50+x)(5000-50x) + 1000 - (5000-50x)25$ in cents.

Prices · # of items · fixed cost · cost of winners

$(5000-50x)[(50+x)-25] + 1000$

$(5000-50x)(25+x) - 1000$

$f(x) = (5000-50x)(25+x) - 1000$

$f'(x) = (25+x)(-50) + (5000-50x) \overset{SET}{=} 0$

$-1250 - 50x + 5000 - 50x$

$100x = 3750$

$x = 37.5$

$f'' = -100 < 0$ ∴ max revenue at $x = 37.5$

cost should be $87\frac{1}{2}$ each, 2 for 1.75

(8) Nupur

let x = # of $5 increases
fare increase

$150 + 5x$ = new fare
$640 - 40x$ = new rides.

$f(x) = (150 + 5x)(640 - 40x)$

$5(10+x)\,40(15-x)$
$200(6+x)(15-x)$
$200(150 + 5x - x^2)$

$3200 + 3200 - 2000x + 3200x - 200x^2$

$f(x) = 3200 + 1200x - 200x^2$

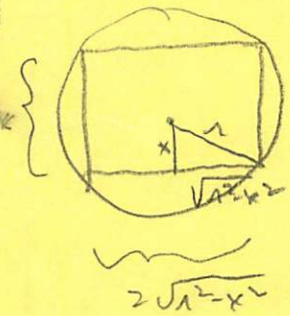
$f'(x) = 1200 - 400x \overset{\text{set}}{=} 0$

$x = 3$

fare need 3 $5 increases
∴ cost should be
$1.65.

$f''(x) = -400$ ∴ max, $f''(x) < 0$

(9)



$A = lw$

$A(x) = 2x \cdot 2\sqrt{\lambda^2 - x^2}$
$= 4x\sqrt{\lambda^2 - x^2}$

$A' = 4\sqrt{\lambda^2 - x^2} + \dfrac{4x(-2x)}{2\sqrt{\lambda^2 - x^2}}$

$A' = \dfrac{4\lambda^2 - 4x^2 - 4x^2}{\sqrt{\lambda^2 - x^2}}$

$= \dfrac{4\lambda^2 - 8x^2}{\sqrt{\lambda^2 - x^2}} \overset{\text{set}}{=} 0$

$\lambda^2 = 2x^2$

$\dfrac{\lambda}{\sqrt{2}} = x$

A'	+	0	−
	0		$\frac{\lambda}{\sqrt{2}}$

max f' test.

max area at $x = \frac{\lambda}{\sqrt{2}}$

$\sqrt{\lambda^2 - \frac{\lambda^2}{2}} = \frac{\lambda}{\sqrt{2}}$

length $= 2\frac{\lambda}{\sqrt{2}} = \frac{2\sqrt{2}}{2}\lambda = \sqrt{2}\lambda$

width $2x = 2\frac{\lambda}{\sqrt{2}} = \frac{2\sqrt{2}\lambda}{2} = \sqrt{2}\lambda$

Since $L = W$, rectangle is a square.

#8

$(150+5x)(640-40x)$

$40.5(30+x)(16-x)$

$200(480-14x-x^2) = f(x)$

$-200(x^2+14x-480) = f(x)$

~~$-200(x+\quad)(x-\quad)$~~

$f'(x) = -200(2x+14) \overset{\text{Set}}{=} 0$

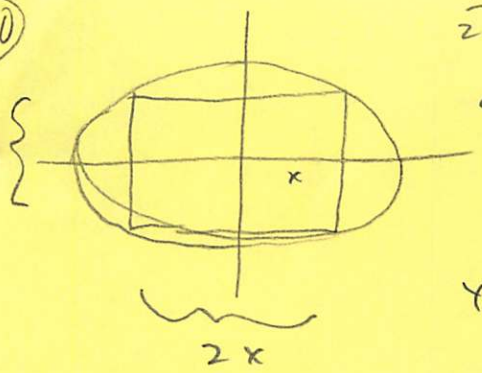
$x = -7$

should reduce the price by

$7.5 = 35¢$

∴ Sales price should be $150 - 35 = \$1.15$

(10)



$\frac{x^2}{25} + \frac{y^2}{9} = 1$

$9x^2 + 25y^2 = 144$

$y^2 = \frac{225 - 9x^2}{25}$

$y = \frac{\sqrt{225-9x^2}}{5}$

$4 = lw$

$A(x) = 2x \cdot \frac{2}{5}\sqrt{225-9x^2}$

$= \frac{4}{5}\left[x\sqrt{225-9x^2}\right]$

$A' = \frac{4}{5}\left[\sqrt{225-9x^2} - \frac{x(9x)}{2\sqrt{225-9x^2}}\right]$

$= \frac{4}{5}\left[\frac{225-9x^2-9x^2}{\sqrt{225-9x^2}}\right] \stackrel{set}{=} 0$

$18x^2 = 225$

$x = \frac{15}{3\sqrt{2}} = \frac{5\sqrt{2}}{2} = \frac{5}{\sqrt{2}}$

A' | + 0 −

0 | $\frac{5}{\sqrt{2}}$

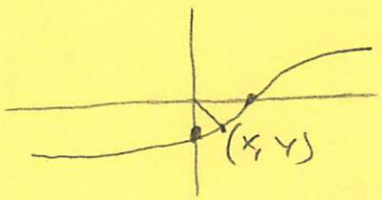
max area $f'(x)$.

Dimensions are $5\sqrt{2} \times 3\sqrt{2}$

$\frac{\sqrt{225 - 9 \cdot \frac{25}{2}}}{5}$ $\frac{1}{5}\sqrt{\frac{15}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

(11)

$y = (3x-4)^{\frac{1}{3}}$



$f(x) = \sqrt{x^2 + (3x-4)^{\frac{2}{3}}}$ distance formula

$f' = \frac{2x + \frac{2}{3}(3x-4)^{-\frac{1}{3}} \cdot 3}{2\sqrt{x^2+(3x-4)^{\frac{2}{3}}}} \neq 0$

$f' = \frac{x + \frac{1}{(3x-4)^{\frac{1}{3}}}}{\sqrt{x^2+(3x-4)^{\frac{2}{3}}}} \stackrel{set}{=} 0$

nearest place is

$\left(\frac{4}{3}, 0\right)$

$(1, -1)$

$x^3(3x-4) = -1$

$3x^4 - 4x^3 = -1$

$3x^4 - 4x^3 + 1 = 0$

$(x-1)^2(3x^2+2x+1) = 0$

	3	−4	0	0	1
1	3	−1	−1	−1	0
1	3	+2	+1	0	

$3x^2 + 2x + 1$

f' | + ⊘ + 4

0 | 1 | $\frac{4}{3}$

Notice f' DNE at $\frac{4}{3}$,

Critical value is not extreme

No endpts, so extreme value must occur at place where f' DNE.

(12)

$f(x) = K\left(\frac{x-9}{x^2} + \frac{1}{100}\right)$

$f'(x) = K\left(\frac{x^2 - (x-9)2x}{x^4}\right)$

$= K\left(\frac{x - 2x + 18}{x^3}\right)$

$= K\left(\frac{18-x}{x^3}\right) \overset{\text{set}}{=} 0$

$x = 18$

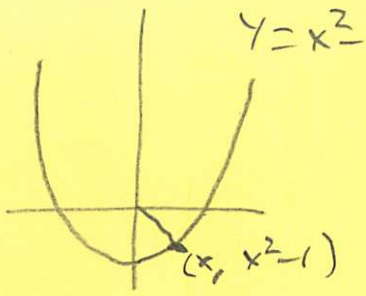
f' $+$ 0 $-$

0 18

max at $x = 18$ f'test

most profitable Price would be \$18 each.

(13)



$f(x) = \sqrt{x^2 + (x^2-1)^2}$

$= \sqrt{x^2 + x^4 - 2x^2 + 1}$

$f(x) = \sqrt{x^4 - x^2 + 1}$

$f'(x) = \frac{4x^3 - 2x}{2\sqrt{x^4 - x^2 + 1}} \overset{\text{set}}{=} 0$

$2x^3 - x = 0$

$x(2x^2 - 1) = 0$

$x \in \{0, \pm\frac{\sqrt{2}}{2}\}$

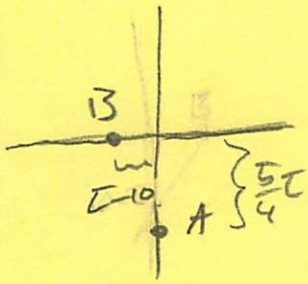
f' $-$ 0 $+$ 0 $-$ 0 $+$

$-\frac{\sqrt{2}}{2}$ 0 $\frac{\sqrt{2}}{2}$

min distance at $x \in \{\pm\frac{\sqrt{2}}{2}\}$

f'test

(14)



Let $\tau = 0$ be 5 pm

Then A's distance from origin in minute is $\frac{75}{60}\cdot\tau$

$A = \frac{5}{4}\tau$

at $\tau = 0$ B is $\tau - 10$ from origin.

$d = \sqrt{\left(\frac{5}{4}\tau\right)^2 + (\tau - 10)^2} =$

$d'(t) = \frac{\frac{5}{4}\cdot\frac{5}{2}\tau + 2(\tau - 10)}{2\sqrt{\left(\frac{5}{4}\tau\right)^2 + (\tau - 10)^2}} \overset{\text{set}}{=} 0$

$\frac{25}{8}\tau + 2\tau - 20 = 0$

$\frac{41}{8}\tau = 20$

$\tau = \frac{160}{41}$

$= 3\frac{37}{41}$

→ 3 min 54 seconds.

so closest at

5:03:54

(15) Let x = tons of coffee per month.

Revenue = $x\left(300 - \frac{x}{1000}\right) - 10x - 1000$

tons, \$ per ton \$10 per ton. tons \$1000

$R(x) = 300x - \frac{x^2}{1000} - 10x - 1000$

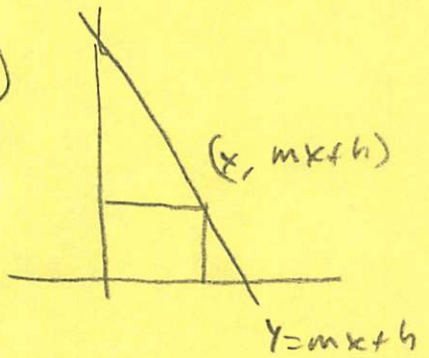
$R(x) = 290x - \frac{x^2}{1000} - 1000$

$R' = 290 - \frac{x}{500} \overset{\text{set}}{=} 0$ $\quad R'' = -\frac{1}{500} < 0$ ∴ max at $x = 145{,}000$ (first)

$x = 145{,}000$

The county should sell 145,000 tons of coffee.

(16)



Note: $m < 0$ in this example

$A_{rect} = lw$

$= x(mx+b)$

$A(x) = mx^2 + bx$

$A'(x) = 2mx + b \overset{\text{set}}{=} 0$

$x = -\frac{b}{2m}$ $\quad y = m\left(-\frac{b}{2m} + b\right)$

$= \frac{b}{2}$

$A''(x) = 2m < 0$ ∴ max at $x = -\frac{b}{2m}$.

$y = mx + b \overset{\text{set}}{=} 0$

$x = -\frac{b}{m}$

$A_\Delta = \frac{1}{2} b \cdot b$

$= \frac{1}{2}\left(-\frac{b}{m}\right) b$

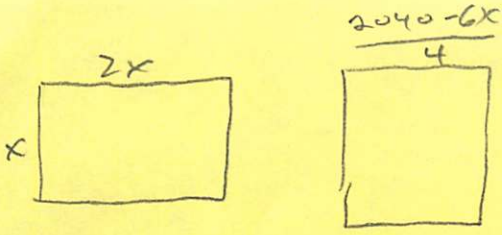
$= \frac{-b^2}{2m}$

$A_{rect} = -\frac{b}{2m} \cdot \frac{b}{2}$

$= -\frac{b^2}{4m}$

max area of rectangle is $\frac{1}{2}$ Area of Triangle.

(17)



$A(x) = 2x^2 + \left(510 - \frac{3}{2}x\right)^2$

$A'(x) = 4x + 2\left(510 - \frac{3}{2}x\right)\left(-\frac{3}{2}\right)$

$4x - 1530 + \frac{9}{2}x \overset{\text{set}}{=} 0$

$\frac{17}{2}x = 1530$

$x = 180$

ENDPTS $x = 0$

Rectangle doesn't exist

Square has sides $\sqrt{510}$ by $\sqrt{510}$.

I recommend NO rectangle field.

$A''(x) = \frac{17}{2} > 0$ min area at $x = 180$ but we want max area.

(18) x = speed.

Cost = wages + fuel

$\$/\text{hr} \cdot \text{hrs}$ $\$/\text{gal} \cdot \text{gal} \cdot \text{hrs}$.

$\text{hrs} = \frac{\text{dist}}{x}$

$C(x) = 11\frac{D}{x} + \frac{5}{4}\left(4 + \frac{x^2}{500}\right) \cdot \frac{D}{x}$

$\frac{D}{x}\left(11 + 5 + \frac{x^2}{400}\right)$

$C(x) = \frac{D}{x}\left(16 + \frac{x^2}{400}\right)$

$C'(x) = -\frac{D}{x^2}\left(16 + \frac{x^2}{400}\right) + \frac{D}{x}\left(\frac{x}{200}\right)$

$D\left[\left(-\frac{16}{x^2} - \frac{1}{400}\right) + \frac{1}{200}\right]$

$D\left(\frac{-16}{x^2} + \frac{1}{400}\right) \overset{\text{set}}{=} 0$

$x^2 = 6400$

$x = 80$

$C''(x) = \frac{32}{x^3} > 0$

$\therefore$ min of C at $x = 80$.

Since the legal speeds are at most 55 mph, I must recommend Driving at the limit at all times.